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TRANSACTION COSTS AND THE SMALL FIRM EFFECT*

Hans R. STOLL

Vanderbilt University, Nashville, TN 37203, USA

Robert E. WHALEY

Vanderbilt University, Nashville, TN 37203, USA

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Recent empirical work by Banz (1981) and Reinganum (1981) documents abnormally large risk-adjusted returns for small firms listed on the NYSE and the AMEX. The strength and persistence with which the returns appear lead both authors to conclude the single-period, two-parameter capital asset pricing model is misspecified. This study (1) confirms that total market value of common stock equity varies inversely with risk-adjusted returns, (2) demonstrates that price per share does also, and (3) finds that transaction costs at least partially account for the abnormality.

1. Introduction

Recent empirical work by Banz (1981) and Reinganum (1981) documents abnormally large risk-adjusted returns for small firms listed on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX). The strength and persistence of the abnormal returns implies either (a) that the market is inefficient, or (b) that the single-period, two-parameter capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Black (1972) on which their empirical tests are based is misspecified, or (c) both.

To conclude that the market is inefficient would be consistent with the widely held view that investors are unjustifiably reluctant to invest in small firms.[1] However, a market is inefficient only if it is possible, on the basis of currently available information, to earn abnormal risk-adjusted returns net of all transaction costs.[2] The Banz and Reinganum studies are based on gross

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[1]Shepherd (1975) and Hall and Weiss (1967), for example, support this view.

[2]Jensen (1978) emphasizes the importance of accounting for *all* costs in tests of market efficiency. Phillips and Smith (1980) observe that many authors ignore transaction costs in tests of the efficiency of the Chicago Board Options Exchange and thereby mistakenly conclude that it is an inefficient market.

returns. A simple explanation of their results is that an investor taking a small firm portfolio position faces higher transaction costs than he does when he takes an otherwise similar large firm position. The market-maker's spread on a proportional basis is generally higher for small firms because of their infrequent trading activity and risk; and the broker's commission rate is, among other things, an inverse function of the price per share, a variable correlated with the total market value of the stock. In addition there are other less explicit costs such as the cost of investigating and monitoring a firm that may be higher for small firms.

Banz and Reinganum, unwilling to reject market efficiency in view of the persistence of their abnormal returns, reject the CAPM. However, they provide no alternative explanation of their results. In this study we show that out-of-pocket transaction costs can at least partially explain the small firm effect. This suggests that transaction costs are a 'missing factor' in the single-period, two-parameter CAPM. It also suggests that, while small firms find it more costly to attract investment funds, unjustified discrimination against small firms is not necessarily present.

In the paper an arbitrage portfolio test methodology is outlined and implemented to replicate the Banz/Reinganum small firm effect and to provide a benchmark for the other tests. The same methodology is then used to show that the small firm effect can also be viewed as a low price effect. We show that the statistical bias induced by infrequent trading of small firms does not appear to explain the small firm effect in the case of the monthly data used in this study. Next, proportional transaction costs are measured and shown to decline with increases in firm size. Finally, the CAPM is assumed to apply to returns net of transaction costs, and the abnormal positive returns of small firms are shown to be eliminated for investment horizons of less than one year. The paper concludes with a summary.

2. Data and test design

The sample consists of NYSE common stocks traded during the period January 1955 through December 1979. Stock prices, shares outstanding, and returns were obtained from the *Center for Research in Security Prices* (CRSP) monthly file, as were the returns of the CRSP equal-weighted and value-weighted market indices. The monthly series of ninety-day prime commercial paper rates for the sample period were compiled from various issues of *Federal Reserve Bulletin.*

To assess the market value effect, 10 portfolios of NYSE stocks were formed. At the beginning of each year T ($T=1960,\ldots,1979$), all stocks with 72 consecutive monthly returns starting 60 months before and ending 12 months after the beginning of year T listed on the CRSP file were ranked in ascending order of total market value of shares outstanding and were clustered into 10 portfolios[3] in the manner of Black, Jensen and Scholes (1972). The 60 months of data prior to T were used to estimate the market model regression,

$$R_{jt}-R_{ft}=\alpha_{jT}+\beta_{jT}(R_{mt}-R_{ft})+\varepsilon_{jt}, \qquad t=T-60,\ldots,T-1, \tag{1}$$

where R_{jt} and R_{mt} represent the realized returns of stock j and the market index in month t, and R_{ft} represents the rate of interest on commercial paper during month t. The coefficient β_{jT} is referred to as stock j's relative risk (i.e., relative to the index employed) for year T. The 12 monthly stock returns during year T were averaged cross-sectionally within each portfolio to obtain a 12-month time series. With the portfolio selection technique applied at the beginning of each of 20 years, 1960 through 1979, 10 portfolio time series of 240 monthly returns (January 1960 through December 1979) were thus generated. The number of stocks in each portfolio averaged approximately 90 and ranged from 71 in 1960 to 116 in 1979.

Not all the NYSE common stocks having 60 months of prior data were included in the sample portfolios. In the 20 cross-sections considered, there were 19,773 stocks which had the 60 consecutive returns preceding the cross-sectional date. Of these, 47 were eliminated because they were delisted at some point during the subsequent 12 months. The excluded firms had an average market value of \$116 million at the beginning of the year and an average monthly return of 3.57 percent during the year. Because the paper examines the effect of transaction costs, it is necessary to have, among other things, the bid–ask spread for the stocks in the sample. To ensure comparability of the no-transaction-cost/transaction-cost results, the stocks included in the portfolios in year T had to have bid and ask prices reported in the final issue of *Stock Quotations on the NY Stock Exchange* by Francis Emory Fitch, Inc., in years $T-1$ and T. The stock's relative spread during the year was taken to be the average of the relative spreads existing at the beginning and at the end of the year. The bid–ask spread constraint eliminated 1,772 stocks whose average market value was \$438 million. The sample, therefore, contained 17,954 stocks in the 20 cross-sections or approximately 90 (i.e., 17,954 ÷ 20 ÷ 10) stocks in each portfolio in each year during the sample period.

3. Portfolio characteristics

Various attributes of the 10 test portfolios are summarized in table 1. The market value ranking procedure produced some interesting results. For

[3]Where the number of stocks, n, did not divide evenly into the 10 portfolios, the $n-\text{int}(n/10)\times 10$ smallest market value portfolios contained $\text{int}(n/10)+1$ stocks. The remainder of the portfolios contained $\text{int}(n/10)$.

Table 1

Mean monthly return, estimated relative risk coefficients, mean total market value, mean price per share, and mean monthly return standard deviation of the NYSE stocks contained in the 10 portfolios arranged in ascending order of total market value (portfolio 1 smallest, portfolio 10 largest) for the years 1960 through 1979.

Portfolio number	Mean monthly portfolio return	Estimated portfolio risk (equal-weighted index)[a]	Estimated portfolio risk (value-weighted index)[a]	Standard deviation of monthly portfolio return	Mean total market value (in millions)	Mean price per share	Mean standard deviation of monthly security returns	Portfolio standard deviation/ Mean security standard deviation
1	0.01603	1.230 (0.030)	1.309 (0.072)	0.07333	$ 14.741	$11.989	0.11328	0.647
2	0.01321	1.113 (0.018)	1.243 (0.055)	0.06397	30.736	18.176	0.10072	0.635
3	0.01131	1.058 (0.015)	1.216 (0.048)	0.06047	50.203	21.141	0.09762	0.619
4	0.01179	0.995 (0.013)	1.173 (0.041)	0.05651	76.873	25.553	0.09250	0.611
5	0.00927	0.939 (0.011)	1.129 (0.035)	0.05322	115.232	29.271	0.08575	0.621

6	0.00910	0.918 (0.012)	1.146 (0.029)	0.05233	172.349	33.678	0.08549	0.612
7	0.00870	0.875 (0.014)	1.126 (0.023)	0.05039	272.349	36.921	0.08197	0.615
8	0.00838	0.824 (0.017)	1.088 (0.022)	0.04837	444.732	38.541	0.07763	0.623
9	0.00705	0.739 (0.019)	1.014 (0.017)	0.04437	729.916	50.772	0.07342	0.604
10	0.00513	0.654 (0.026)	0.985 (0.012)	0.04263	3347.513	65.021	0.06779	0.629
Mean	0.01000	0.934	1.143	0.05456	525.511	33.106	0.08762	0.622

	Mean	Standard deviation
Equal-weighted CRSP return	0.00997	0.05559
Value-weighted CRSP return	0.00666	0.04219
Prime commercial paper rate	0.00492	0.00189

[a]Relative risk coefficients estimated with the market model regression, $R_{pt}-R_{ft}=\alpha_p+\beta_p(R_{mt}-R_{ft})+\varepsilon_{pt}$. Values in parentheses are the standard errors of the relative systematic risk coefficients.

example, the mean realized returns of the portfolios over the 240-month time series decrease monotonically as the total market values of the stocks in the portfolios increase. The lowest market value portfolio, portfolio 1, yielded approximately 1.6 percent a month, while the highest market value portfolio, portfolio 10, yielded 0.5 percent a month. Thus, before adjusting for risk, the smallest firms outperformed the largest firms by more than 13 percent annually.

The estimated relative risk coefficients of the portfolios, $\hat{\beta}_p$, obtained by regressing the monthly portfolio risk premiums on the monthly market risk premiums,

$$R_{pt}-R_{ft}=\alpha_p+\beta_p(R_{mt}-R_{ft})+\varepsilon_{pt}, \qquad t=1,\ldots,240, \tag{2}$$

also decrease monotonically with market value, independent of which market index was used. The value-weighted index, however, yields risk estimates greater than 1 for all but the largest portfolio. Since small firms are typically more volatile than large firms, the finding that the equal-weighted test portfolios are on average more risky than the value-weighted market proxy is not surprising. If the mean and standard deviation of the value-weighted index, 0.666 percent and 4.219 percent, are compared to those of the equal-weighted index, 0.997 percent and 5.559 percent, the effect is even more clearly dramatized.

The results of the regressions using the equal-weighted index are more intuitively appealing, yielding estimates of β_p both above and below one. If the test portfolios and the market index are both equal-weighted and if the test portfolios contain all of the stocks in the market index, the average portfolio realtive risk would be equal to 1. The average value of 0.934 reflects that data constraints systematically eliminated some of the smaller, more volatile firms from the 10 portfolios.

The mean total market value figure reported in table 1 is the mean of the mean beginning-of-year market values of the stocks in each portfolio during the 20-year sample period. Note that although portfolio 10 contains approximately the same number of stocks as the other portfolios it has over 63 percent of the market's total value.

The mean price per share and the mean standard deviation on monthly stock returns are included in the table to illustrate that the 'market value effect' may be a misnomer. Evident in the table is a strong correlation between price per share and market value and an inverse correlation between standard deviation of return and market value. Conditioning returns on these variables may also produce abnormal returns. Furthermore, the fact that these variables appear in studies of the bid–ask spread [e.g., Tinic (1972), Stoll (1978b)] gives credence to the hypothesis that transaction costs are a missing factor in the single-period CAPM.

The final column, portfolio return standard deviation divided by mean stock return standard deviation, is included to demonstrate that an investor can achieve the same level of relative diversification in each market value category. Equal dollar investments in the stocks of portfolio 1, for example, reduce risk to 0.0733, 64.7 percent of the average risk of the component stocks. At the other extreme, in portfolio 10 the portfolio risk is reduced to 62.9 percent of the average risk of its component stocks, nearly the same percentage.

4. The small firm effect

To compare the returns of the portfolios with one another, arbitrage portfolio return series are generated. At the beginning of each year T ($T=1960,\ldots,1979$), the stocks within each market value decile are ranked in ascending order of estimated relative risk and divided into two equal-weighted portfolios. Where the number of stocks is odd, the high-risk portfolio contains an additional stock. One dollar is then allocated between the low-risk and the high-risk portfolios such that the overall market value portfolio has a relative risk coefficient equal to 1. Finally, the equal-weighted market index returns[4] are subtracted and the arbitrage portfolio returns are formed.[5] Note that the net investment in each arbitrage portfolio and the net relative risk are zero.

Since the arbitrage portfolios are formed using beginning-of-year estimated relative risk coefficients, it is possible that, due to the regression phenomenon, the arbitrage portfolio will not have zero relative risk during the year. To purge any remaining market effects, the arbitrage portfolio return series are regressed on the market risk premium series in the manner of Black and Scholes (1974, p. 17),

$$R_{at}=\alpha_a+\beta_a(R_{mt}-R_{ft})+\varepsilon_{at}. \tag{3}$$

[4]Unless otherwise specified, subsequent references to market returns refer to the CRSP equal-weighted index returns. This study, like those of Black, Jensen and Scholes (1972), Fama and Macbeth (1973), Black and Scholes (1974), Banz (1981) and Reinganum (1981), employs test portfolios which are equal-weighted combinations of stocks. With such the case, it is most appropriate to use an equal-weighted market index as the benchmark portfolio. [See Brown and Warner (1980, pp. 239–243).] The analysis was also carried out using a value-weighted market index, and the conclusions were unchanged.

[5]This arbitrage portfolio test methodology is essentially the same as that employed by Watts (1978). The Banz (1981) procedure was also employed. With it, the monthly portfolio returns are levered or unlevered at the riskless rate so as to adjust the portfolio relative risk level to 1. The market returns are then subtracted from each of the unlevered/levered time series to obtain 10 costless arbitrage portfolio series.

A problem with this second approach is that, in general, it is necessary to borrow at the riskless rate to adjust the large firm portfolio risk to 1. If the commercial paper rate understates the rate at which investors can borrow, the risk-adjusted return of the large firm portfolios are overstated and, hence, the small firm/large firm return differential is reduced.

The intercept terms from these regressions are estimates of the abnormal returns realized by engaging in the arbitrage activity.

Table 2 contains the abnormal returns and their Student t-ratios for the entire 20-year sample period and for various subperiods. For the entire sample period, portfolio 1 yielded an abnormal return of 0.53 percent a month, compared with -0.47 percent a month for portfolio 10. On an annualized basis, small firms outgained large firms by about 12 percent during the period 1960 through 1979 after adjustment for risk. This figure is very close to the Banz (1981, p. 15) estimate of 12.12 percent (i.e., $1.01\% \times 12$), although his value is computed for the period 1931 through 1975 and his portfolios contain fewer securities.[6]

The results are fairly consistent across the various subperiods. The abnormal returns are distributed around zero, with the small firm portfolios performing above average and the large firm portfolios performing below average. It is worthwhile to note that the mean abnormal return across portfolios is not necessarily equal to zero. This is attributable to two effects. First, the portfolios contain only a subset of the stocks used in the CRSP equal-weighted index. Due to the exclusion criteria imposed in data selection, this subset generally consists of larger, more stable stocks, as is reflected by the mean relative risk of 0.934. (See table 1.) Holding all other factors constant, this would tend to make the mean abnormal return positive. Second, the portfolios are formed annually on the basis of historical relative risk estimates. The estimates generally regress toward unity during the calendar year so that the estimated slope coefficient in (3) will generally be non-zero and the estimated intercept term, $\hat{\alpha}_a$, will be different from the mean arbitrage portfolio return, $\bar{R}_a$.

5. Is the small firm effect due to statistical bias?

The closing price for a stock in a given day is the price recorded at the time of the last transaction. In the case of frequently-traded stocks, the last transaction is likely to occur near the instant at which the market closes for the day. On the other hand, in the case of infrequently-traded stocks, the last transaction may well have occurred some time beforehand. This phenomenon imposes two sources of bias on the relative risk coefficients estimated using historical returns based on closing prices. First, as noted by Fisher (1966), the market return on a given day is a mixture of stock returns for that day and for previous days, so that the market return series will be positively autocorrelated and, hence, the estimated market return variance will be

[6]The Banz estimate was realized by hedging the risk-adjusted returns of the 50 smallest NYSE stocks against the risk-adjusted returns of the 50 largest. The number of stocks in each of our 10 test portfolios averages 90.

Table 2

Mean monthly abnormal returns on the 10 arbitrage portfolios arranged in ascending order of the total market value of the common stock.[a]

$$R_{at} = \alpha_a + \beta_a(R_{mt} - R_{ft}) + \varepsilon_{at}$$

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$\hat{\alpha}_6$	$\hat{\alpha}_7$	$\hat{\alpha}_8$	$\hat{\alpha}_9$	$\hat{\alpha}_{10}$
Overall period										
1960–1979	0.0053	0.0028	0.0014	0.0020	−0.0004	−0.0004	0.0004	0.0011	−0.0026	−0.0047
	(3.49)	(3.03)	(1.77)	(3.10)	(−0.74)	(−0.63)	(−0.49)	(−0.99)	(−2.14)	(−3.23)
10-year periods										
1960–1969	0.0052	0.0024	0.0006	0.0006	−0.0007	−0.0011	−0.0008	−0.0003	−0.0019	−0.0041
	(2.70)	(2.12)	(0.58)	(0.64)	(−0.88)	(−1.38)	(−0.86)	(−0.29)	(−1.50)	(−2.59)
1970–1979	0.0055	0.0031	0.0021	0.0032	−0.0003	0.0002	0.0000	−0.0018	−0.0034	−0.0054
	(2.34)	(2.10)	(1.64)	(3.76)	(−0.29)	(0.22)	(0.02)	(−0.99)	(−1.68)	(−2.23)
5-year subperiods										
1960–1964	0.0030	0.0018	−0.0006	0.0005	−0.0010	0.0002	0.0012	0.0019	−0.0009	−0.0020
	(1.24)	(1.21)	(−0.40)	(0.42)	(−0.91)	(0.24)	(0.98)	(1.45)	(−0.57)	(−0.94)
1965–1969	0.0075	0.0031	0.0017	0.0006	−0.0003	−0.0025	−0.0029	−0.0025	−0.0030	−0.0063
	(2.56)	(1.80)	(1.24)	(0.48)	(−0.30)	(−1.96)	(−2.00)	(−1.49)	(−1.51)	(−2.81)
1970–1974	0.0027	0.0018	0.0014	0.0032	0.0004	0.0019	0.0022	0.0010	0.0003	0.0023
	(0.78)	(0.78)	(0.71)	(2.41)	(0.27)	(1.26)	(1.26)	(0.38)	(0.12)	(0.66)
1975–1979	0.0056	0.0018	0.0021	0.0034	−0.0009	0.0005	−0.0010	−0.0026	−0.0045	−0.0116
	(1.71)	(1.07)	(1.18)	(2.87)	(−0.70)	(0.32)	(−0.51)	(−1.04)	(−1.62)	(−3.45)

[a]To form the arbitrage portfolio return series, the stocks within each market value decile were ranked in ascending order of relative risk and divided into two equal-weighted portfolios. One dollar was then allocated between the low-risk and the high-risk portfolios so as to adjust the overall relative risk level to 1. The portfolio return series was then formed and the market return series was subtracted, with the resulting series representing returns on portfolios requiring no net investment and having no relative risk. Values in parentheses are Student t-ratios for the null hypothesis $H_0\!:\alpha_a=0$.

downward biased. Second, since the estimated covariance between the stock's and the market's returns focuses only on contemporaneous (and not on leading and lagged) association(s), it too will be downward biased. The net effect of these biases will be downward biased relative risk estimates for stocks traded less frequently than the typical stock in the market index and upward biased estimates for stocks traded more frequently than the typical stock in the market index.[7]

Roll (1981) argues that since there is a strong relation between frequency of trading and firm size, the market value anomaly may be solely attributable to this estimation bias. If small firms have underestimated relative risk coefficients and large firms have overestimated relative risk coefficients, the risk-adjusted performance of the small firms will obviously appear better than large firms, holding all other factors constant. When daily return data are used, the bias is likely to be severe and is, as Roll points out, reflected in the results of Reinganum (1981). When monthly return data are used, however, the bias should be substantially reduced.

To evaluate the degree of bias in the relative risk coefficients estimated using monthly return data, the regression

$$R_{pt} - R_{ft} = \alpha_p + \beta_{p1}(R_{mt-1} - R_{ft-1}) + \beta_{p2}(R_{mt} - R_{ft}) + \beta_{p3}(R_{mt+1} - R_{ft+1}) + \varepsilon_{pt}, \qquad t = 2, \ldots, 239, \tag{4}$$

was run for each of the 10 portfolios. Dimson (1979) demonstrates that when stocks are subject to infrequent trading the sum of the lagged, contemporaneous, and leading coefficients in a regression such as (4) provides an unbiased estimate of the stock's relative risk.[8] The parameter estimates for each of the regressions were aggregated and the results are reported in table

[7]This result was first pointed out by Black and Scholes (1973), and was later pursued by Scholes and Williams (1977) and Dimson (1979).

[8]The selection of a model that includes only 1 lagged and 1 leading variable was somewhat arbitrary. A model including 2 lagged and 2 leading variables was also tested, however, none of the added variables had coefficients which were significantly different from 0 at the 5 percent level.

Some additional tests were also performed. Dimson (1979, p. 205) suggests that the appropriate number of lagged and leading variables to include in the market model regression may be determined by comparing the cross-sectional variance of the relative risk estimates for the kth lag, $\mathrm{VAR}(\hat{\beta}_k)$, with the cross-sectional mean of the estimated variances of the risk estimates, mean$[\mathrm{var}(\hat{\beta}_k)]$. If the difference between the values is approximately equal to 0, then the kth coefficient may be omitted from the model specification. For each year in the period 1955 through 1980, the market model regressions including lagged and leading returns were estimated for all stocks on the CRSP monthly return file which had the most recent 60 months of return data, and the difference between $\mathrm{VAR}(\hat{\beta}_k)$ and mean $[\mathrm{var}(\hat{\beta}_k)]$ was calculated for each lag k. In general, the difference was only positive for lag 0 (i.e., the contemporaneous market return) and was indistinguishable from 0 for all other lags. In other words, the relative risk estimates obtained by regressing monthly stock returns on the contemporaneous market returns are not seriously biased as a result of the infrequent trading phenomenon.

3. To facilitate comparison, the simple linear regression coefficients from table 1 are also included. While the aggregated coefficients of portfolio 1 and portfolio 10 move in the direction predicted by Roll, the difference between the aggregated coefficient and the simple regression estimate is always less than 0.06 and is less than 0.03 in 9 of the 10 portfolios.

To illustrate that this magnitude of difference is of little consequence to the market value anomaly, consider the values reported for the risk and the return of portfolio 10 (i.e., the worst case) in tables 1 and 3. The abnormal return for a portfolio may be stated as

$$(\bar{R}_p - \bar{R}_f)(1/\hat{\beta}_p) - (\bar{R}_m - \bar{R}_f).$$

On the basis of the data in table 1, the increase in abnormal return as a result of substituting the 'corrected' value of relative risk, 0.601, for the simple regression estimate, 0.654, is 0.00003. Clearly this is insufficient to explain the observed difference in abnormal returns of small and large firms. Thus, the market value effect demonstrated in table 2 appears to be robust to Roll's estimation bias criticism.[9]

Table 3

Estimated relative risk coefficients of the 10 monthly portfolio return series (portfolio 1 smallest, portfolio 10 largest) for the period January 1960 through December 1979.

Portfolio number	Estimated relative risk[a] $\hat{\beta}_p$	Standard error	Aggregated coefficient[b] $\hat{\beta}_{p1}+\hat{\beta}_{p2}+\hat{\beta}_p$	Standard error	Difference between risk estimates $\hat{\beta}_p-(\hat{\beta}_{p1}+\hat{\beta}_{p2}+\hat{\beta}_{p3})$
1	1.230	0.030	1.249	0.048	−0.019
2	1.113	0.018	1.109	0.028	0.004
3	1.058	0.015	1.052	0.024	0.006
4	0.995	0.013	1.003	0.019	−0.008
5	0.939	0.011	0.953	0.017	−0.014
6	0.918	0.012	0.945	0.020	−0.027
7	0.875	0.014	0.865	0.023	0.010
8	0.824	0.017	0.803	0.028	0.021
9	0.739	0.019	0.748	0.031	−0.009
10	0.654	0.026	0.601	0.043	0.053

[a]Relative risk coefficients estimated with the market model regression $R_{pt}-R_{ft}=\alpha+\beta_p(R_{mt}-R_{ft})+\varepsilon_{pt}$, $t=1,\ldots,240$.

[b]Aggregated coefficient obtained by summing the regression coefficients, $\hat{\beta}_{p1}+\hat{\beta}_{p2}+\hat{\beta}_{p3}$, from the regression $R_{pt}-R_{ft}=\alpha_p+\beta_{p1}(R_{mt-1}-R_{ft-1})+\beta_{p2}(R_{mt}-R_{ft})+\beta_{p3}(R_{mt+1}-R_{ft+1})+\varepsilon_{pt}$, $t=2,\ldots,239$.

[9]Reinganum (1982) finds that the use of daily data produces a substantial bias in estimates of relative risk, as suggested by Roll. However the magnitude of the bias is too small to explain the small firm effect.

6. The low price effect

The implication of the results reported in tables 1 and 2 is that an investor can earn abnormally high returns by investing in small firms. The preliminary evidence in table 1 and the significance of price per share in determining transaction costs suggest that the same effect may be found for stocks with low prices. To test this proposition the experiment was repeated using price per share as the stratification variable: (a) stocks were ranked from lowest to highest according to price per share and were clustered into deciles; (b) stocks within each decile were ranked according to relative risk and were divided into two equal-weighted portfolios; (c) one dollar was allocated between the low-risk and high-risk portfolios such that the overall price per share decile had a relative risk coefficient equal to 1; and, finally, (d) the arbitrage portfolio return series were generated and the regressions (3) were carried out. The abnormal returns of the portfolios were reported in table 4.

The portfolio consisting of the lowest priced stocks dominated the portfolio containing the highest priced stocks by about ten percent annually during the overall sample period. While the magnitude of the difference is slightly less than the magnitude of the difference in the market value tests reported in table 2, the pattern with which the abnormal returns decrease with an increase in the price per share is the same in the overall period as well as in the various subperiods. It is interesting to note that Blume and Husic (1973) find a similar low price effect in a regression of market model residuals against a price variable, and, in a recent paper, Miller and Scholes (1981) discover that the inverse of price per share is significantly positively related to mean excess return.[10]

7. Transaction costs

Ignoring transaction costs, it appears that an investor can earn abnormal returns by concentrating his investments in low market value or low price per share stocks. However, to enact either of these trading strategies an investor is likely to face significant out-of-pocket transaction costs, including both the dealer's bid–ask spread and the broker's commission.

The dealer stands ready to buy shares immediately at the bid price or to sell shares immediately at the ask price. The cost to the dealer of providing this liquidity arises from the risk of holding an inventory, from clerical costs,

[10]The stock's standard deviation of return was also used as a stratification variable and the analysis was replicated. In this case there was no systematic relationship between abnormal portfolio return and average standard deviation of return of the portfolio stocks. This implies that the relative risk estimate used in calculating abnormal returns adequately captured the effect of the standard deviation.

Table 4

Mean monthly abnormal returns on the 10 arbitrage portfolios arranged in ascending order of the price per share of the common stock.[a]

$$R_{at} = \alpha_a + \beta_a(R_{mt} - R_{ft}) + \varepsilon_{at}$$

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$\hat{\alpha}_6$	$\hat{\alpha}_7$	$\hat{\alpha}_8$	$\hat{\alpha}_9$	$\hat{\alpha}_{10}$
Overall period										
1960–1979	0.0063	0.0024	0.0012	0.0015	0.0001	−0.0001	−0.0008	−0.0016	−0.0022	−0.0021
	(3.09)	(2.28)	(1.80)	(2.32)	(0.22)	(−0.10)	(−0.88)	(−1.70)	(−1.75)	(−1.27)
10-year subperiods										
1960–1969	0.0056	0.0014	−0.0003	0.0012	−0.0004	−0.0006	−0.0016	−0.0009	−0.0017	−0.0021
	(2.43)	(1.27)	(−0.37)	(1.55)	(−0.48)	(−0.62)	(−1.73)	(−1.02)	(−1.48)	(−1.38)
1970–1979	0.0074	0.0036	0.0027	0.0017	0.0006	0.0005	0.0000	−0.0024	−0.0029	−0.0024
	(2.24)	(2.04)	(2.66)	(1.63)	(0.61)	(0.42)	(−0.02)	(−1.45)	(−1.33)	(−0.85)
5-year subperiods										
1960–1964	0.0025	0.0005	−0.0004	0.0028	−0.0003	0.0012	0.0009	−0.0009	−0.0004	−0.0015
	(0.84)	(0.33)	(−0.30)	(3.01)	(−0.25)	(1.05)	(0.66)	(−0.81)	(−0.29)	(−0.79)
1965–1969	0.0087	0.0023	−0.0003	−0.0003	−0.0005	−0.0024	−0.0040	−0.0009	−0.0029	−0.0028
	(2.57)	(1.49)	(−0.22)	(−0.28)	(−0.41)	(−1.68)	(−3.38)	(−0.65)	(−1.72)	(−1.15)
1970–1974	0.0054	0.0025	0.0034	0.0014	0.0008	0.0006	0.0030	0.0004	0.0001	0.0028
	(1.12)	(0.92)	(2.13)	(0.88)	(0.57)	(0.35)	(1.40)	(0.17)	(0.03)	(0.68)
1975–1979	0.0038	0.0021	0.0010	0.0017	0.0003	0.0012	−0.0009	−0.0028	−0.0027	−0.0043
	(0.87)	(0.94)	(0.75)	(1.28)	(0.20)	(0.78)	(−0.46)	(−1.27)	(0.88)	(−1.11)

[a]To form the arbitrage portfolio return series, the stocks within each price per share decile were ranked in ascending order of relative risk and divided in two equal-weighted portfolios. One dollar was then allocated between the low-risk and the high-risk portfolios so as to adjust the overall relative risk level to 1. The portfolio return series was then formed and the market return series was subtracted, with the resulting series representing returns on portfolios requiring no net investment and having no relative risk. Values in parentheses are Student t-ratios for the null hypothesis $H_0\!:\alpha_a = 0$.

and from the losses he incurs in trading with more informed traders.[11] Empirical studies of the proportional bid–ask spread have shown than it varies inversely with price per share and a measure of trading activity, such as volume, and varies directly with a measure of risk such as return variance.[12] The broker, an agent for the investor, seeks the best price, executes transactions, and maintains records. He is compensated by a commission which, as a proportion of the price, is a decreasing function of price per share and number of round lots traded.

Data were collected for these out-of-pocket costs to determine whether differences according to firm size can explain the small firm effect. Needless to say there are other, less explicit, costs of trading that have been ignored and that may vary by size of firm. In particular one thinks of the costs of becoming informed about the firm and of monitoring its activities.

Bid and ask prices were collected for each NYSE stock for the last trading day of each year from Fitch's *Stock Quotations on the NY Stock Exchange*. The proportional spread, calculated as

$$\frac{\text{ask price} - \text{bid price}}{(\text{ask price} + \text{bid price})/2},$$

represents compensation to the dealer on a turn-around transaction (purchase and sale).[13] On a single transaction, the cost to the investor is one-half the spread.

The commission rate on a transaction was computed from the minimum commission rate schedule available in various issues of the *NYSE Fact Book*. Prior to December 1968, the commission as a proportion of transaction value depends only on price per share and is independent of the number of shares traded. Subsequently the proportional commission is a function of price per share and number of shares traded. Given a transaction value and a stock price, the proportional commission paid on the transaction can be estimated.[14] For each year, the dollar transaction value used for all stocks

[11]For theoretical work along these lines, see Tinic (1972), Stoll (1978a) and Ho and Stoll (1981). The effect of informed as opposed to liquidity traders was first presented in Bagehot (1971).

[12]The first empirical study of the determinants of the market-maker's bid–ask spread, Demsetz (1968), did not consider a risk measure. More recent studies by Tinic (1972), Tinic and West (1972), Benston and Hagerman (1974), Tinic and West (1974), Grant and Whaley (1978) and Stoll (1978b) have generally found it to be a significantly positive determinant.

[13]In those cases in which the *Stock Quotations on the N.Y. Stock Exchange* (the 'Fitch sheet') was not available for the last trading day of the year, the last day of the year for which the Fitch sheet was available was used.

[14]The procedure for calculating commissions is as follows. At the beginning of each year, the stock's price was divided into the average dollar transaction size for the year and the quotient was rounded to the nearest 100. The product of the 'rounded' number of transacted shares and the price per share was assumed to be the amount of money involved in the transaction, and the

was the average dollar transaction size in that year as derived from data in the *NYSE Fact Book*.[15] For the period since the introduction of negotiated rates in May 1975, the last minimum commission rate schedule is assumed to apply.[16]

To illustrate the relationship between market value and transaction costs during the sample period, the mean percentage spread and the mean percentage commission rate on the stocks within each portfolio in each year were computed. To approximate the costs of trading the equal-weighted and the value-weighted market portfolios, equal-weighted and value-weighted averages of the stock transaction cost rates were also computed. The results are reported in table 5.

In general, both the relative spread and the commission rate decrease as the market value of the stocks in each portfolio increases. In the former case, the effect is pronounced, decreasing from an overall mean relative spread of 2.93 percent for portfolio 1 to 0.69 percent for portfolio 10. With the commission rates, the effect is less dramatic. For the smallest firms the commission rate on a turnaround transaction averaged 3.84 percent, while for the largest firms the turnaround commission averaged 2.02 percent. The total turnaround transaction cost differential is quite substantial (averaging $3.84+2.93-2.02-0.69=4.06\%$), apparently reflecting the higher costs to brokers and dealers of completing transactions in stocks of small firms. The economies of trading large firms are also reflected in the transaction cost difference for the equal-weighted and value-weighted indices. Also notable in table 5 is the increase in proportional commission rates in the period 1960 to 1975. While in part due to a decline in stock prices, it also reflects surcharges imposed prior to the introduction of negotiated rates.

8. Is there a small firm effect when returns are measured net of transaction costs?

Gross returns used in the earlier tests of the small firm effect are computed on the basis of the prices reported for the last transactions in consecutive months (with adjustments for cash dividends, if any were paid during the month). Since the closing transaction of the month is a purchase or a sale

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round lot commission schedule was applied to determine the dollar commission charge. The commission rate was then computed by dividing the commission charge by the money involved in the transaction. By ignoring odd lots, there is a slight downward bias in the calculated commission.

[15]The average dollar transaction size in 1970, for example, was $12,750.

[16]Since May 1975, commission rates have declined on large transactions and have risen on small transactions (of full service brokers). The effect on the level of commission rates for the transaction sizes used in the study is not clear. An analysis of the structure of commissions as related to price per share, based on data from a few full service brokers and two discount brokers, suggests that, as before, commissions are a decreasing function of price per share.

Table 5

Mean percentage transaction cost rates (relative spreads and commissions) of the NYSE common stocks contained in the 10 portfolios arranged in ascending order of total market value of the stock (portfolio 1 smallest, portfolio 10 largest) for the years 1960 through 1979.[a]

Year		Portfolio number										Equal-weighted market index	Value-weighted market index
		1	2	3	4	5	6	7	8	9	10		
1960	Relative spread	3.23	2.30	1.99	1.62	1.52	1.36	1.38	1.19	1.28	0.73	1.66	0.85
	Commission rate	1.59	1.32	1.29	1.12	1.00	1.00	0.96	1.05	0.81	0.69	1.08	0.75
1961	Relative spread	3.11	2.23	1.96	1.65	1.50	1.33	1.26	1.24	1.11	1.07	1.65	0.89
	Commission rate	1.79	1.47	1.43	1.20	1.14	1.12	1.08	1.03	0.98	0.81	1.21	0.85
1962	Relative spread	2.86	2.18	1.77	1.56	1.42	1.21	1.19	1.10	1.03	1.07	1.54	0.85
	Commission rate	1.64	1.41	1.34	1.15	1.00	0.99	0.95	0.91	0.80	0.75	1.10	0.74
1963	Relative spread	3.39	2.24	1.58	1.62	1.56	1.19	1.15	1.08	0.86	1.10	1.58	0.84
	Commision rate	1.92	1.54	1.34	1.28	1.19	1.10	1.03	1.00	0.90	0.88	1.22	0.86
1964	Relative spread	2.88	1.77	1.57	1.33	1.17	1.05	1.05	0.86	0.76	0.64	1.31	0.63
	Commission rate	1.87	1.47	1.28	1.24	1.16	1.09	0.97	0.98	0.85	0.84	1.18	0.80
1965	Relative spread	2.05	1.51	1.36	1.08	1.02	0.92	0.93	0.80	0.70	0.60	1.10	0.58
	Commission rate	1.75	1.40	1.30	1.16	1.11	1.04	0.95	0.95	0.86	0.80	1.13	0.78
1966	Relative spread	2.19	1.89	1.34	1.15	1.06	1.09	1.01	0.86	0.75	0.66	1.20	0.67
	Commission rate	1.59	1.28	1.20	1.11	1.05	1.02	0.90	0.89	0.83	0.76	1.07	0.77
1967	Relative spread	1.96	1.88	1.36	1.23	1.24	1.13	0.99	0.95	0.85	0.64	1.22	0.69
	Commission rate	1.70	1.46	1.40	1.21	1.20	1.09	1.03	0.98	0.94	0.82	1.18	0.97
1968	Relative spread	1.69	1.30	1.20	1.27	1.09	1.00	0.94	0.92	0.86	0.59	1.09	0.66
	Commission rate	1.45	1.25	1.19	1.14	1.05	0.94	0.95	0.93	0.90	0.73	1.05	0.79
1969	Relative spread	1.95	1.56	1.35	1.34	1.24	1.19	1.12	0.96	0.96	0.67	1.24	0.77
	Commission rate	1.26	1.16	1.10	1.03	0.99	0.93	0.94	0.90	0.85	0.77	0.99	0.83
1970	Relative spread	2.41	2.02	1.64	1.48	1.41	1.33	1.09	1.05	0.93	0.65	1.40	0.74
	Commission rate	1.68	1.56	1.43	1.38	1.31	1.24	1.21	1.17	1.10	0.96	1.30	1.03

1971	Relative spread	2.62	1.93	1.63	1.33	1.34	1.11	1.06	0.93	0.79	0.62	1.34	0.72
	Commission rate	1.90	1.62	1.47	1.40	1.37	1.25	1.23	1.19	1.11	0.96	1.35	1.03
1972	Relative spread	2.56	1.72	1.39	1.31	1.11	1.03	0.89	0.83	0.91	0.53	1.23	0.67
	Commission rate	1.54	1.39	1.33	1.32	1.26	1.22	1.21	1.21	1.13	1.01	1.26	1.06
1973	Relative spread	3.05	2.11	1.88	1.49	1.41	1.22	1.13	0.95	1.04	0.54	1.48	0.66
	Commission rate	1.85	1.60	1.59	1.52	1.47	1.43	1.40	1.40	1.34	1.08	1.47	1.13
1974	Relative spread	4.69	3.27	2.57	2.34	1.83	1.72	1.47	1.27	1.07	0.66	2.09	0.79
	Commission rate	2.56	2.08	1.96	1.91	1.81	1.76	1.65	1.65	1.59	1.31	1.83	1.37
1975	Relative spread	5.08	3.20	2.66	2.02	1.73	1.51	1.37	1.21	0.89	0.66	2.04	0.74
	Commission rate	3.23	2.42	2.21	2.00	1.89	1.81	1.76	1.73	1.62	1.50	2.02	1.48
1976	Relative spread	3.37	2.25	1.65	1.36	1.27	1.08	0.94	0.87	0.73	0.52	1.41	0.58
	Commission rate	2.52	2.10	1.83	1.75	1.71	1.65	1.62	1.60	1.52	1.38	1.77	1.42
1977	Relative spread	2.89	1.71	1.41	1.32	1.09	0.98	0.88	0.82	0.72	0.53	1.24	0.58
	Commission rate	2.21	1.80	1.68	1.66	1.60	1.57	1.53	1.51	1.49	1.36	1.64	1.40
1978	Relative spread	3.23	1.93	1.64	1.43	1.24	1.12	1.00	0.94	0.78	0.62	1.39	0.67
	Commission rate	2.22	1.80	1.64	1.64	1.58	1.56	1.52	1.53	1.50	1.44	1.64	1.46
1979	Relative spread	3.33	1.97	1.68	1.43	1.36	1.16	1.09	0.90	0.83	0.64	1.44	0.70
	Commission rate	2.17	1.70	1.61	1.53	1.52	1.48	1.48	1.45	1.43	1.36	1.57	1.39
Mean	Relative spread	2.93	2.05	1.68	1.47	1.33	1.19	1.10	0.99	0.89	0.69	1.43	0.71
	Commission rate	1.92	1.59	1.48	1.39	1.32	1.26	1.22	1.20	1.13	1.01	1.35	1.04

[a]The relative spreads are the average of the average beginning- and end-of-year values of (ask − bid)/(bid + ask)/2 for each of the stocks within each of the portfolios. The bid and ask prices were recorded from *Stock Quotations on the N.Y. Stock Exchange* by Francis Emory Fitch, Inc. The commission rate on each stock was computed from the minimum commission schedule provided in various issues of *NYSE Fact Book*. Effective May 1, 1975 all commissions became subject to negotiation between member firms and their customers. For the years 1975 through 1979, the 1974 schedule was applied.

with equal probability, the prices used in the return computation are, on average, half way between the bid and the ask prices, and the reported return is an unbiased estimate of the realized before-transaction-cost holding period returns.

To evaluate the effect of trading costs on the market value anomaly, the two-parameter CAPM is assumed to apply to after-transaction-cost returns. The 10 portfolios formed earlier on the basis of total market value are used. The monthly returns of each of the stocks in each of the portfolios during the period January 1960 through December 1979 were adjusted to incorporate transaction costs by applying the formula

$$R^{\tau}_{jt}=(1+R_{jt})(1-F_{jt})/(1+F_{jt})-1, \qquad t=1,\ldots,240, \tag{5}$$

where R^{τ}_{jt} denotes the after-transaction-cost rate of return on stock j in month t, R_{jt} is the before-transaction-cost rate of return, and F_{jt} is the proportional transaction cost for stock j during month t, obtained by summing one half the proportional bid–ask spread for the stock and the commission rate for the stock.[17]

With the monthly stock returns adjusted for transaction costs and aggregated into 10 portfolios as in the before-transaction-cost analysis, the portfolio returns were generated so as to adjust the portfolio relative risk coefficients to 1. The returns of the equal-weighted market index, *net of the equal-weighted market transaction costs*, were subtracted from the portfolio return series, and then the arbitrage portfolio returns were regressed on the after-transaction-cost market returns. The abnormal performance measures from the regressions are reported in table 6.

The results indicate that the market value effect is reversed when the transaction costs are considered. The largest firms outperform the smallest firms by about seventeen percent a year during the period 1960 through 1979. At the five percent level, the abnormal returns of portfolios 1 through 3 are significantly negative. At the other extreme, the large firm portfolios have positive abnormal returns, although not always significant. It appears that once trading costs are incorporated, the market value anomaly continues to exist, but in the opposite direction with small firms having negative abnormal holding period returns.

9. The low price effect and transaction costs

The final test involved imposing transaction costs on the returns of the

[17]Note that the adjustment (5) assumes all income from the stock is earned through price appreciation.

Table 6

Mean monthly after-transaction-cost abnormal returns on the 10 arbitrage portfolios arranged in ascending order of the total market value of the common stock.[a]

$$R^{\tau}_{at} = \alpha_a + \beta_a(R^{\tau}_{mt} - R_{ft}) + \varepsilon_{at}$$

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$\hat{\alpha}_6$	$\hat{\alpha}_7$	$\hat{\alpha}_8$	$\hat{\alpha}_9$	$\hat{\alpha}_{10}$
Overall period										
1960–1979	−0.0136	−0.0052	−0.0031	0.0001	−0.0003	0.0014	0.0023	0.0028	0.0012	0.0004
	(−7.70)	(−4.87)	(−3.26)	(0.16)	(−0.41)	(1.76)	(2.37)	(2.26)	(0.86)	(0.25)
10-year subperiods										
1960–1969	−0.0130	−0.0047	−0.0027	−0.0003	−0.0001	0.0008	0.0020	0.0037	0.0036	0.0024
	(−5.61)	(−3.52)	(−2.33)	(−0.33)	(−0.15)	(0.86)	(1.69)	(2.93)	(2.31)	(1.28)
1970–1979	−0.0145	−0.0053	−0.0028	0.0010	−0.0003	0.0021	0.0027	0.0017	−0.0010	−0.0014
	(−5.33)	(−3.10)	(−1.89)	(0.98)	(−0.29)	(1.62)	(1.74)	(0.82)	(−0.40)	(−0.48)
5-year subperiods										
1960–1964	−0.0232	−0.0084	−0.0035	−0.0010	0.0005	0.0016	0.0066	0.0068	0.0088	0.0092
	(−7.88)	(−4.53)	(−1.89)	(−0.62)	(0.35)	(1.27)	(4.23)	(4.09)	(4.37)	(3.54)
1965–1969	−0.0050	−0.0019	−0.0018	−0.0001	−0.0007	−0.0003	−0.0017	0.0011	−0.0003	−0.0026
	(−1.55)	(−1.01)	(−1.27)	(−0.04)	(−0.56)	(−0.24)	(−1.08)	(0.61)	(−0.16)	(−1.04)
1970–1974	−0.0168	−0.0099	−0.0050	0.0000	−0.0007	0.0054	0.0058	0.0067	0.0071	0.0109
	(−4.11)	(−3.51)	(−2.08)	(−0.01)	(−0.36)	(2.92)	(2.62)	(1.99)	(1.95)	(2.53)
1975–1979	−0.0132	−0.0023	−0.0013	0.0018	−0.0001	0.0001	0.0005	−0.0016	−0.0065	−0.0105
	(−3.66)	(−1.26)	(−0.69)	(1.46)	(−0.04)	(0.03)	(0.24)	(−0.62)	(−2.19)	(−2.92)

[a]To form the arbitrage portfolio return series, the stocks within each market value decile were ranked in ascending order of relative risk and divided into two equal-weighted portfolios. One dollar was then allocated between the low-risk and the high-risk portfolios so as to adjust the overall relative risk level to 1. The after-transaction-cost portfolio return series was then formed and the after-transaction-cost market return series was subtracted, with the resulting series representing returns on portfolios requiring no net investment and having no relative risk. Values in parentheses are Student t-ratios for the null hypothesis H_0:$\alpha_a = 0$.

stocks in the portfolios formed on the basis of price per share. The arbitrage portfolio selection methodology was the same as before, and culminated with the market model regressions of the arbitrage portfolio returns on the returns of the market index.[18] The reversal of the anomaly was similar to the test using market value as the stratification variable. For the overall period the portfolio consisting of the highest priced stocks had an abnormal return of 0.82 percent per month and the portfolio consisting of the lowest priced stocks experienced an abnormal return of -1.54 percent per month. Both returns were significant at the 5 percent level. On an annualized basis, the highest priced stocks outperformed the lowest priced stocks by more than 28 percent.

10. Investment horizon

The results in table 6 reveal that concentrating investment in low market value stocks does not produce positive abnormal returns after transaction costs. During the overall sample period 1960 through 1979, the strategy involving investment in the lowest market value decile of NYSE yielded an after-transaction-cost abnormal return of -1.36 percent a month. The use of monthly returns and equal-weighted test portfolios implies that investors incur transaction costs each month on all their stocks. Use of a longer holding period would tend to reduce the negative abnormal returns of small firms. Indeed there is some holding period for which the results of Banz and Reinganum would reappear, albeit the reduction in number of observations implied by an increase in the holding period would tend to lessen the statistical significance of the results.

To check on the effect of changing the holding period, the arbitrage portfolio methodology was repeated using 2-month, 4-month, 6-month and 12-month holding period returns. Included in this procedure was a calculation of individual stocks' relative risk coefficients using returns defined over to the desired holding period length. The before-transaction-cost and after-transaction-cost abnormal returns of the small firm portfolio for the various investment horizons are reported in table 7. For convenience, the figures for the 1-month holding period are also reported. Note that the before-tranaction-cost abnormal returns of the small firms are less than the monthly abnormal return times the length of the holding period. For example, the 12-month abnormal holding period return of 5.99 percent is less than 12 times the 1-month abnormal return (i.e., $12 \times 0.53\% = 6.36\%$). This reflects, in part, the fact that the holding period return is a geometric sum of

[18]The test results using price per share as the stratification variable are available from the authors.

Table 7
Mean abnormal returns on the lowest total market value arbitrage portfolio[a] for various investment horizons during the period January 1960 through December 1979.

Investment horizon (in months)	Number of time series observations	Before-transaction-cost abnormal return[b]	After-transaction-cost abnormal return[c]
1	240	0.0053 (3.49)	−0.0136 (−7.70)
2	120	0.0088 (2.80)	−0.0093 (−2.91)
3	80	0.0129 (2.36)	−0.0058 (−1.09)
4	60	0.0190 (2.61)	0.0004 (0.06)
6	40	0.0266 (2.36)	0.0069 (0.65)
12	20	0.0599 (2.15)	0.0453 (1.75)

[a]To form the arbitrage portfolio return series, the stocks within the lowest market value decile were ranked in ascending order of relative risk and divided into equal-weighted portfolios. One dollar was then allocated between the low-risk and the high-risk portfolios so as to adjust the overall relative risk level to 1. The portfolio return series was then formed and the market return series was subtracted, with the resulting series representing returns on portfolios requiring no net investment and having no relative risk. The values in parentheses are Student t-ratios for the null hypothesis $H_0\!:\alpha_a=0$.

[b]The before-transaction-cost abnormal return refers to the estimated intercept parameter in the regression $R_{at}=\alpha_a+\beta_a(R_{mt}-R_{ft})+\varepsilon_{at}$.

[c]The after-transaction-cost abnormal return refers to the estimated intercept parameter in the after-transaction-cost regression $R^{\tau}_{at}=\alpha_a+\beta_a(R^{\tau}_{mt}-R_{ft})+\varepsilon_{at}$.

monthly returns, which, given the variability of returns, is less than the arithmetic sum.[19]

As expected, the after-transaction-cost abnormal return for the small firm portfolio does become positive as the investment horizon is increased. The 'break-even' holding period, at which the after-transaction-cost abnormal return is zero, is about 4 months for the small firm portfolio (i.e., the abnormal return is 0.04%). In other words, at a holding period of 4 months the after-transaction-cost performance of a risk-adjusted portfolio of small firms is equal to the after-transaction-cost performance of the equal-weighted market index. However, none of the positive after-transaction-cost abnormal

[19]A similar attenuation is observed in the difference between before-transaction-cost abnormal returns of the smallest and largest firms. For holding periods of 1, 2, 3, 4, 6 and 12 months these differences, with the Student t-ratios in parentheses, are 0.0100(3.81), 0.0189(3.55), 0.0282(3.04), 0.0378(3.21), 0.0478(2.52), 0.0884(2.02). All are significant at the 5 percent level except the 12-month holding period return for which the critical t-value at 20 observations is 2.086.

returns, even for the 12-month holding period, is statistically significant at the 5 percent level. The negative after-transaction-cost abnormal returns for the 2-month holding period, −0.93 percent, as well as for the 1-month holding period reported earlier, −1.36 percent, are statistically significant.[20]

The procedure followed in this study has, in effect, been to test the joint hypothesis of market efficiency and the single-period CAPM as applied to net returns after transaction costs. On the basis of a 1-month holding period, the null hypothesis is rejected at the 5 percent level. The small firm portfolio earns a significantly negative abnormal return after transaction costs during the 20-year sample period. As the length of the investment horizon is increased, the effect is diminished; and the after-transaction-cost abnormal returns for the small firm portfolio become positive (as found by Banz and Reinganum). However, these returns are not statistically significant, even for a holding period of 1 year.

11. Summary

This study re-examines the small firm effect which seems to have garnered widespread attention. Recent studies by Banz and Reinganum demonstrate that investors can consistently earn positive before-transaction-cost abnormal returns by investing in small firms. For this reason they conclude that the two-parameter CAPM is misspecified. A test of the CAPM, however, is also a test of market efficiency. The appropriate question to ask in this type of analysis is whether investors can earn abnormal returns net of all transaction costs by trading on the basis of the market value of the stock. Based upon the empirical evidence provided herein, the answer to the question is contingent upon the length of the investment horizon. For an investment horizon of one month, the mean abnormal return for the small firm portfolio is significantly negative. However, for investment horizons between three months and one year, the abnormal returns are not significantly different from zero. Thus, the data are consistent with the CAPM applied to after-transaction-cost returns defined over these longer investment horizons.

[20]For holding periods of 1, 2, 3, 4, 6 and 12 months the differences in after-transaction-cost abnormal returns of the smallest and largest firms are −0.0140(−4.59), −0.0056(−1.01), 0.0048(0.52), 0.0137(1.20), 0.0245(1.36), 0.0782(1.91). The numbers in parentheses are the Student t-ratios.

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