Levered and inverse ETPs: Blessing or curse?

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ABSTRACT

Levered and inverse ETPs are designed to provide geared long and short exposures to the daily returns of different benchmark indexes. The benchmarks can be any reference index. The popular ones are on stocks, bonds, commodities and volatility. The problem with these products is that they are not generally well-understood, particularly those whose benchmarks are futures-based indexes. They are neither suitable buy-and-hold investments nor effective hedging tools. They are unstable and exist only as a mechanism for placing short-term directional bets. Levered and inverse products are not, and cannot be, effective investment management tools.

KEYWORDS

Levered and inverse funds, geared investments, front-running, futures indexes, contango

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Levered ETPs are again in the news. Citigroup's 3*x* Long Crude Oil ETN (UWT) nearly tanked on March 9, 2020 when its benchmark index dropped 73.5%, just shy of the 75% acceleration clause which would have triggered termination. On March 15, 2020, ProShares announced the liquidation of its UltraPro 3*x* Crude Oil (OILU) and UltraPro –3*x* Crude Oil (OILD) ETFs. While the 91.1% year-to-date decline of OILU was certainly basis for liquidation, OILD had risen 252.1%. Presumably the liquidation of OILD was not based on an acceleration event or shareholder considerations, but rather on the natural hedging that occurs in the futures replication of OILU and OILD. Invoking acceleration clauses, however, is nothing new. Two years earlier, Credit Suisse closed its Daily Inverse VIX Short Term ETN (XIV) after it lost 96.3% of its value in a single day. Naturally, these types of events are confusing to most and undermine investor confidence in U.S. financial markets. Fact of the matter is that the value-destroying characteristics of these products are not well-understood. In November 2019, the SEC proposed new regulations rules that would require brokers and advisers to ask their clients a series of questions before they could sell them leveraged ETPs.[i] Client responses would presumably help advisers determine whether investors understand all of the risks inherent in the products.[ii] The problem is inherently much deeper than investor education. Why do the products exist?

The main attraction of geared levered and inverse funds is that they offer an inexpensive, convenient, highly-levered, limited-liability means for profiting from a directional price view. If an investor has a strong view that the stock market will rise over next few weeks, an S&P 500 ETF such as UPRO, ProShares UltraPro S&P 500 (3*x*), or SPXL, Direxion Daily S&P 500 Bull 3X Shares (3*x*), might be suitable. Trading costs and barriers to entry are low. Bid/ask spreads on these products are trivial for active ETPs, and commission-free brokerage, with no account minimums, is now the norm.[iii] While a similar price risk exposure can be obtained in the CME Group's S&P 500 futures market, trading costs are higher and entering the market may not be infeasible. Retail investors, for example, may not be able to open futures accounts because of account and personal wealth minimums. But, by buying shares of UPRO or SPXL on margin, the retail customer can quickly and cheaply achieve 6*x* leverage on the S&P 500 index. Similarly, some institutional investors are blocked. Many states, for example, limit the use of derivatives for public pension funds. Leveraged and inverse funds circumvent the rules. While the Tennessee Consolidated Retirement System allows the use of stock index futures, they may not be used for purposes of "speculative leveraging" or exceed 10% of the market value of the fund's total assets.[iv] New York's Common Retirement Fund limits the use of index futures to "rebalancing objectives."[v] Since leveraged and inverse ETPs are classified as equity securities as opposed to derivatives, the risk management rules do not apply.

The most important problem with geared levered and inverse funds is that most of them are expected to collapse. The longevity of a levered or inverse ETP depends critically on the expected return and volatility of the fund's benchmark index. Cheng and Madhavan (2009) (hereafter "CM") show analytically that, (a) if the expected return of the benchmark index is positive, inverse and levered inverse funds will eventually fail (i.e., their expected values are zero), and (b) if the expected benchmark return is negative, long and levered long funds will fail. We extend these results in three important ways. First, we demonstrate how the expected life of an ETP can be estimated using Monte Carlo simulation and provide actual estimates for many of the most actively traded funds. Our simulation of a 3*x* natural gas ETF, for example, shows its life can be as few as 63 days and has a median of slightly more than a year (398 days). Second, CM dismiss their negative expected index return result by saying that it "…seems unlikely in a long-run equilibrium, but it is included for completeness."[vi] This is understandable. At the time, levered and inverse funds on commodity and volatility futures indexes were in their infancy. Now, they are among the most popular and the most controversial. Indeed, all of the recent fiascos mentioned in the opening paragraph are levered and inverse ETPs benchmarked to futures indexes. Futures-based indexes assume the idiosyncrasies of the futures contract prices upon which they are based. In markets such as crude oil, natural gas, and volatility, long hedging demand generally exceeds short hedging demand. When this happens, the futures price must rise to a level high enough for speculators to step in, sell futures to absorb the hedging demand imbalance, and earn a fair risk premium. Because the speculator sells the futures, the equilibrium expected futures return must be negative. We show, through simulation, that both highly levered long <u>and</u> short funds on such futures indexes are expected to fail,[vii] and that ETPs on such futures indexes are expected to collapse even if they have no leverage!

Finally, the realized daily ETP return is different from its levered benchmark return for a variety of reasons including management fees, licensing fees, operating costs, contract indivisibilities, basis risk or slippage, and front-running. The difference between the daily return of the ETP and the daily return of the benchmark is called *tracking error,* and is useful in assessing the degree to which issuers have achieved their stated investment objectives.

With all of this by way of backdrop, we now focus on the economics, mechanics, and empirics of levered and inverse funds—not because they provide opportunities to invest in new asset classes or manage the risk of investment portfolios in an effective way, but rather because their inherent instability in volatile markets has led to financial market disarray and rapid-fire fund liquidations. We provide an appraisal of what these funds are intended to do, and why and when they can be expected to fail.

History of Levered and Inverse Funds

The evolution of exchange-traded products (ETPs) in the U.S. has been fast-paced. The first generation of products is characterized by the fact that they hold that securities that constitute the benchmark index. The benchmark may be stocks, bonds, or commodities like gold. They are classified as exchange-traded funds (ETFs) because their specific holdings are fully transparent on a daily basis. The first ETF launched in the U.S. was SPY, the SPDR S&P 500 ETF Trust, in January 1993. The chief innovation of second generation ETPs is that they are fully collateralized futures positions. In place of holding portfolios of securities, the fund holds T-bills (i.e., the collateral) and an <u>equal</u> notional amount of futures. Many are classified as ETFs because they report their holdings each day, listing the T-bills and the specific futures contract positions. USCF's U.S. Oil Fund, USO, was the first to appear in April 2006 and is long a nearby monthly crude oil futures contract. The nearby contract is rolled into the second nearby during the second week of the delivery month. ProShares' VIX Short-Term Futures ETF, VIXY, was launched in January 2011 and represented the first VIX ETF. Barclays was actually the first to launch a VIX ETP. It did so two years earlier in January 2009. VXX, the iPath Series A S&P 500 VIX Short-Term Futures ETN has a different structure. It is an exchange-traded note (ETN) that promises to deliver the return of the S&P 500 VIX Short-term Futures Index, which, itself, is a fully collateralized VIX futures position. Due to its ETN structure, Barclays does not have to disclose exactly how they replicate the benchmark index. They may do so in any manner including managing its risk with other types of volatility positions on its books. The only position that eliminates basis risk, however, is the one that replicates the index holdings—in this case, a dynamically rebalanced portfolio of VIX futures.

The third generation of ETPs are geared levered and inverse funds. To understand levered and inverse funds, a single descriptor, *Lx*, is needed, where *L* is the leverage ratio (or gear). Where L=1, the promised fund return is the return of the benchmark itself. Thus, the USO, VIXY, and VXX funds described above are 1*x* funds. If the fund is $L = -1$, it is an inverse or short fund and promises a return of –100% that of the benchmark index. Note that neither the 1*x* or –1*x* funds are levered. Moreover, a –1*x* fund is fully collateralized. The notional amount of futures matches the dollar amount of the T-bills, with the difference being, instead of buying (going long) the futures, it sells (goes short). Leverage applies where the absolute value of *L* is greater than 1 (i.e., $|L| > 1$). 2*x* refers to a two-times long fund, and –2*x* refers to a two-times inverse or short fund, and so on. Note that these funds are under-collateralized. The amount of the T-bills remains the same. The futures positions, however, are twice as large as they were before. Technically, 2*x* products can be referred to as levered long funds, and –2*x* products as levered inverse or short funds. Funds with leverage ratios between –1 and +1 (e.g., 0.5*x*) are over-collateralized funds. The notional amount of the futures position is less than the dollar amount of T-bills.

ProShares was the pioneer in the levered and inverse product space. In June 2006, they launched collateralized futures ETFs with 2*x* and –1*x* leverage and inverse ratios on several major U.S. stock indexes including the S&P 500, the QQQ, the Dow 30, and the S&P 400. In July 2006, they followed up with –2*x* ETFs on the S&P 500 and Dow 30, and two days later, with –2*x* ETFs on the QQQ and S&P 400. The first challenger to ProShares dominance in the levered equity ETF space was by Direxion in November 2008, at which time they launched 3*x* and –3x ETFs on the S&P 500, the Russell 2000, and certain sector indexes. ProShares was also the first-mover in the levered and inverse bond space. In April 2008, they launched –2*x* ETFs on the Barclay's 20+ year and 7-10 year Treasury bond indexes. Direxion followed suit in April 2009 with 3x and –3x ETFs on the same bond indexes. Following a series of business acquisitions, the benchmark indexes are now named the ICE U.S. Treasury 20+ Year and 7-10 Year Bond Indexes. In the commodity space, the major players are ProShares (with ETFs) and Credit Suisse (with ETNs). Here, most of the trading volume is in the levered and inverse products on crude oil and natural gas. For crude oil products, ProShares launched $2x$ and $–2x$ ETFs in November 2008, and Credit Suisse launched $3x$ and $–3x$ ETNs in February 2012. Curiously, the Credit Suisse ETNs were delisted in December 2016, with Citibank filling in the void almost immediately with identical product structures. For natural gas products, ProShares launched $2x$ and $–2x$ ETFs in November 2011, and Credit Suisse launched $3x$ and $–3x$ ETNs in February 2012. Finally, in the volatility space, the two major players are Credit Suisse and Barclays, in descending order of AUM. Credit Suisse was the first-mover with the controversial TVIX $2x$ ETN in November 2010. They introduced the equally controversial XIV ($–1x$) ETN at the same time. ProShares launched the ETF counterparts, UVXY and SVXY, in October 2011.

Current Products

To provide an understanding of the scope of the ETP market in the U.S., we turned to ETFdb.com to identify a complete listing of active funds as of March 13, 2020. Table 1 provides a summary by issuer. Many things stand out. First, there are currently 2,330 ETPs listed on stock exchanges in the U.S., with total assets under management (AUM) of $4,231.6 billion. Second, BlackRock (iShares), Vanguard, and State Street account for 25.2% of the number of issues and 80.9% of the AUM, dominant market positions to say the least. Third, none of the top three issuers have levered and inverse funds. Neither does Schwab, First Trust, VanEck, nor WisdomTree. Presumably these industry ETP industry leaders made early business decisions to avoid levered and inverse products because of their complexity and propensity to provide surprise investor outcomes. Fourth, of the 2,330 ETPs, 277, or 11.9%, of total ETPs are levered and inverse funds. They account for only 1.2% of total AUM. While the percent of total AUM may seem small, its ability to generate revenue is formidable. With the total AUM for levered and inverse funds at $50.6 billion and a typical expense ratio of about 100 basis points, total revenue is on order of $506 million a year.

Table 2 slices the levered and inverse ETP data another way—by asset class. By far, the largest single category is equities with AUM of $42 billion, about 83.2% of the total. Apparently, investors relish the opportunity to lever long and short stock market risk exposures. The bets are clearly short-term. The turnover rate is 16.9%, meaning the shares turnover every 6 days or so. Commodities and volatility products are next largest by AUM, with market shares of 6.1% and 5.0%, respectively. Here the turnover rates are even higher—51.3% and 114.1%, respectively. Indeed, the volatility products are being day-traded. Their holding period (i.e., the inverse of the turnover rate) is 0.88, less than a day! The top three asset classes in Table 2 account for nearly 95% of AUM, and short-term betting is the primary trading motive.

Expected Levered and Inverse Fund Performance

The performance of levered and inverse funds can be addressed in two ways—expected and actual. We turn first to expected fund performance, and we do so in an environment in which there are no management fees, operating expenses, and the like. There is only a benchmark index and the geared levered and inversed products on that benchmark. The two key elements in this discussion are the expected benchmark return and the compounding mechanics associated with leveraging.

Expected benchmark returns

Cheng and Madhavan (2009) derived two important results regarding levered and inverse funds that are relevant to this study. Under standard assumptions regarding asset price movements, they show that: (a) if the expected return on the benchmark index is positive, the long-run value of an inverse or levered inverse fund is zero, and (b) if the expected return on the benchmark index is negative, the long-run value of leveraged long funds is zero. CM downplay the importance of the second result because the levered and inverse funds that existed back in 2009 were almost exclusively largely based on security indexes. The only futures-based products were ProShares 2*x* and –2*x* crude oil ETFs, and they were not launched until November 2008. With the CM's focus on security indexes, it is commonly accepted that long-run equilibrium index returns are expected to be positive. Indeed, the Sharpe (1964)-Lintner (1965) CAPM states that the expected return of a risky security should exceed the risk-free rate. Securities indexes such as the S&P 500, Russell

2000, and 20+ year U.S. Treasury bonds satisfy this criterion. We call these *carry markets* because there is active arbitrage activity between the securities markets and the futures market. In the absence of costless arbitrage opportunities, the futures price will be at full carry and reflect the difference between the risk-fee interest rate and the income rate on the underlying security. The futures market is critical to providing the necessary gearing of the levered and inverse funds.[viii]

The reality is, of course, that levered and inverse commodity and volatility funds have gained a strong toehold. For these benchmark indexes, long-run equilibrium returns are not determined by the CAPM relation, but by an equilibrium established between hedgers and speculators in the relevant futures market. The key economic framework is that of Keynes (1930). He argues that, in commodity futures markets, the price relation between the futures and the underlying security/commodity is not actively arbitraged like it is for stocks and bonds. These *non-carry markets* occur because the cost of delivery of the underlying asset is high and effective short selling is difficult, if not impossible. Without active arbitrage, the futures price curve becomes a series of market clearing futures prices at the different contract expirations. In Keynes' model, there are three sets of traders. Short hedgers are those who sell futures to hedge price risk. Consider a farmer, for example, who as a matter of routine, hedges the crop he seeds in the spring by selling fall wheat futures in order to lock in the price at which he can sell his crop at harvest. On the other side of the trade may be a long hedger, who buys futures to lock in the price at which he can buy wheat—a breakfast cereal producer, perhaps, who has locked in short term sales contracts to grocers. In this example, short hedging demand is larger than long hedging demand. What happens? Speculators step in to pick up the imbalance–but wait until the futures price falls below the expected spot price at the futures expiration date. In this example, the expected return of the futures contract over its life is positive. Keynes calls this futures market equilibrium *normal backwardation*, a condition common to agricultural futures markets because short hedging demand exceeds long hedging demand.

The markets for crude oil, heating oil, and VIX futures typically display an opposite condition called *contango*. Short-term futures prices are usually higher than expected spot prices because long hedging demand exceeds short hedging demand. Speculators step in only when the futures price gets high enough that they can earn a fair risk premium by selling the futures. In the crude oil market, the buying demand is in large part airlines who need to hedge the cost of fuel because its ticket sales have locked in revenue. In the natural gas market, the largest buyers are power companies who need to hedge input costs in the production and sale of electricity. In the volatility market, the largest buyers are stock portfolio managers who want to hedge tail risk. In all cases, the typical futures price curve is steeply upward sloping in the short term. Speculators sell, but at prices above the expected future spot price. The difference between the futures price and the expected spot price is the speculator's *risk premium*. In equilibrium, futures returns are expected to be negative to compensate the speculator for risk-bearing.

<u>Return compounding mechanics</u>

The *compounding effect* arises because the holding period return actually delivered or the compounded daily levered return (i.e., *CLR* on the left-hand side of eqn. (1)) is different from what is expected or the levered compounded return (i.e., *LCR* on the right-hand side of eqn. (1)), that is,

$$CLR = \prod_{t=1}^{T} \left(1 + LR_t\right) - 1 \neq L\left[\prod_{t=1}^{T}\left(1+R_t\right) - 1\right] = LCR \tag{1}$$

where L is the leverage ratio, R_t is the daily benchmark index return on day t, and T is the holding period expressed in number of days. This relation is confusing to retail investors, and justifiably so. Consider the following scenario. An investor buys a –2*x* fund and plans to hold it for two days (i.e., $L = 2$ and $T = 2$). The benchmark return ends up being 5% on day one and –5% on day two. Thus, the two-day benchmark return is –0.25%, the investor, however, expects that the return of his –2*x* fund to be 0.50%. The reality, of course, is that the two-day fund return is –1.00%. The investor not only receives 1.50% less than expected, but also finds the two-day return to be the opposite sign! The <u>only</u> instances in which the two sides of the equation are equal is where the holding period is a single day, $T = 1$, or the leverage ratio is one, $L = 1$.

Eqn. (1) is also the basis for saying levered and inverse ETPs are <u>not</u> effective hedging instruments for any holding period greater than one day. The reason is obvious. To be an effective hedge over a *T*-day hedging interval, the hedge return needs to be equal to *LCR*. If a university endowment holds U.S. stock index funds as its equity allocation and wants to hedge stock market risk temporarily—perhaps, as the ramifications of the Coronavirus become more fully understood, selling S&P 500 futures offers an effective one-to-one, passive hedge. Ideally, an inverse S&P 500 ETF should do the same thing, but it does not. The *CLR* is unpredictably different from the needed hedge return, and the difference widens the longer the holding period and the greater the stock market volatility. Of course, the inverse ETF hedge could be rebalanced dynamically in such a way that it would be effective, but that defeats the purpose. Rebalancing costs would be prohibitive. The only (uninteresting) case in which the levered ETP is an effective passive hedge over more than one day is one in which the investor is hedging a position that is equal and opposite of the proposed hedge in a separate trading account.

To develop a clear understanding of the practical implications of the performance of the levered and inverse products, we turn to modelling the problem numerically using Monte Carlo simulation. Monte Carlo simulation is an underused analysis tool, ideal for application in contexts such as this. It allows us to consider thousands of possible paths that the benchmark index may take over the life of a fund, rather than just <u>one</u> (i.e., the one that has been realized historically). We assume that log returns are normally distributed with mean μ and standard deviation σ. We estimate these parameters for six benchmark indexes using daily data from the period December 20, 2005 through March 13, 2020. We use a common sample period for all indexes so that realized returns/volatilities are compared on an apples-to-apples basis. The newest index is S&P 500 VIX Short-term Futures Index, which began reporting on December 20, 2005. The index level data was downloaded from Bloomberg.

The first three indexes are total return cash indexes in carry markets. SPY and RTY are equity indexes and correspond to the total returns of the S&P 500 and Russell 2000, respectively. IDCOT20T is a bond index and corresponds to the total return of the ICE U.S. Treasury 20+ Year Bond Index. In equilibrium, the carry indexes should have positive expected returns. This means that, over long histories, they should have positive realized returns. The daily returns for our 14½ year sample period confirm this. Table 3 shows that the SPX grew at a compound annual growth rate (CAGR) of 7.87% with an annualized volatility of 19.59%. Similarly, the CAGR for RTY is 5.70% with a return volatility of

24.53% for RTY, and the CAGR for IDCOT20T is 7.69% with a return volatility of 14.11%.

The second three indexes are excess return futures indexes in non-carry markets. The crude oil index, SPGSCLP, is created from the returns of the nearby monthly crude oil futures contract.[ix] As time passes, the contract approaches its delivery date. On the fifth day of the delivery month, the index begins a five-day roll period. In the first of the roll period, 20% of the nearby contract is replaced with the second nearby contract, 20% on the second day, and so on over the remaining three days. The natural gas index, SPGSNGP, is created is the same manner using the returns of the nearby monthly natural gas futures contract. The VIX futures index is based on the returns of the two nearby VIX futures and is rebalanced each day to maintain a constant one month to expiration. Each day, a proportion of the nearby futures contract is sold and replaced with the second nearby futures.[x] Both index constructions represent long, short-term futures contracts and are, therefore, vulnerable to the effects of contango. In equilibrium, such non-carry markets have negative expected returns. Over long histories, we expect to see negative realized returns. This is exactly what we do see over our 14½ year sample period. Table 4 reports that the crude oil index, SPGSCLP, grew at a CAGR of –15.51% with an annualized volatility of 36.58%, the natural gas index, SPGSNGP, grew at –35.94% with volatility of 44.90%, and the volatility index, SPVXSP, grew at –40.82% with volatility of 67.07%.

Each panel in Tables 3 and 4 contains the simulation results of a single benchmark security index. A simulation run is 5,040 trading days (20 years) in length. Based on the historical CAGR and volatility in the top row of the panel, daily index levels are generated. Then, based on these index levels, daily holding period returns are computed. The daily holding period returns, in turn, are used to compute the daily returns of the different levered products. Six leveraged products are considered, with integer values ranging from –3 to +3. In each run, the product's life ends on the day when the fund's value falls below 5% of its initial level.[xi] We use 10,000 simulation runs. Before turning to the results, it is probably worth repeating that the simulation results are conducted in a perfect world with no fees, costs, or other trading impediments. The finite lives of these products in these simulations are driven solely by the expected return/volatility characteristics of the benchmark and the compounding mechanics.

Turning to the results of Table 3, we focus first on Panel A: SPX, the stock market poster child. The row labelled "Life in days" reports the minimum and median of the life of the fund under different leverage assumptions. Note that, where $L=1$, the minimum number of days is 5,040. Recall that the two sides of (1) are equal where $L=1$. Looking down that column, none of the simulation runs ended before 20 years had elapsed. The probability that the life is less than 20 years is zero. This makes sense. The stock market has never fully collapsed. Now, consider the levered inverse fund –3*x*. The median life of the fund is 1,644 trading days, or about 6½ years, and its life can be as little as 439 days. These SPX results are consistent with the CM prediction that negative leverage funds on benchmarks with positive expected returns will eventually collapse. Even at the other extreme, high positive leverage, fund collapse is possible. Consider $L=3$. The median fund life is 5,040, reflecting the fact that most of the simulation runs did not lead to fund collapse within the 20-year period. But, 7.6% of the runs had early termination, and one run lasted only 853 days.

The rest of the panel quantifies the range of differences between the compounded levered return *CLR* (i.e., what is actually earned on the levered fund) and levered compounded return *LCR* (i.e., the return the retail investor expects). The rows headed "*CLR*" contains information about the compounded levered return. The rows headed "*LCR*–*CLR*" is the difference between the levered compounded and the compound levered return. Finally, the last row in the panel reports the frequency with which the *CLR* had a different sign than the *LCR*, a particularly unintuitive result. Focusing on the SPX results first, the panel shows that the return differential is 0 where $L=1$. We noted this earlier. The two sides of eqn. (1) are equal where the fund is not levered. For the $L=-3$ fund, the investor earned as much as 26.5% less than expected and had returns with opposite signs 4.4% of the time. At $L=3$, the investor earned as much as 10.1% less than expected and had returns with opposite signs 1.9% of the time.

The results for the Russell 2000 total return index and the ICE U.S. Treasury 20+ Year Bond Index are qualitatively similar to SPY. An additional insight brought about by the results, however, is that the range of results varies directly with the volatility of the benchmark. The median life of the different levered funds is longest for IDCOT20T where volatility is 14.11%, and shortest for RTY where volatility is 24.53%. RTY's volatility is higher than SPX since it represents a mid-cap versus large-cap benchmark.

The non-carry market simulation results reported in Table 4 are much more extreme. Recall that these have negative expected return because the futures price curves in the natural gas, crude oil, and volatility markets are typically in contango. Here, the leverage and compounding effects behave in some unexpected ways. Where CM predict that the positive leverage will funds will collapse, the simulation results indicate that highly levered long and highly levered inverse funds will collapse. The crude oil results, reported in Panel A of Table 4, shows that the 3*x* fund has a median life of 859 days, can have a life as few as 123 days, and is certain to collapse within 20 years. Even at $L=1$, there is a 66.8% chance of collapse. Now, consider the levered inverse funds. At $L=-1$, the situation is most stable, with only a 2.9% chance that the fund will fail. This makes sense considering we are taking an unlevered short position in a benchmark index that has a negative expected return. But, at $L=-3$, there is an 87.7% chance of collapse. The median life is about 10 years, and the life can be as few as 168 days.

The results for natural gas in Panel B and volatility in Panel C are even more dramatic. Both have expected returns lower and volatility rates higher than crude. A 3*x* natural fund, for example, has an expected life of about one year, and can be as little as 63 days. A 3*x* volatility fund has an expected life of 253 days, and can be as little as 41 days. The probability of these 3x funds not having a life of 20 years is 100%. With the large negative expected returns, the –3*x* funds are expected to last modestly longer. For volatility, however, the chances are 99.9% of failure within the 20-year horizon.

In sum, the Monte Carlo simulations indicate that levered and inverse funds on commodities and volatility will have a value below 5% of their initial value within a few years. What should be disconcerting to investment professionals is that this type of analysis could have been performed before any commodity and volatility products were launched. Monte Carlo simulation is an invaluable investment analysis tool.

The simulations use a 5% stopping criterion to represent the maximum loss an investor is willing to tolerate before liquidating his investment in the fund. As a practical matter, the reason that levered and inverse ETP prices never actually reach zero is that issuers engage in periodic reverse splits (ostensibly to avoid liquidation and preserve the management fee revenue stream). A reverse stock split is an action taken by a corporation (or, in this case, an ETP issuer) to inflate the share price. This is done by consolidating the number of shares by a factor such as 2 or 5 in the case of a 1-for-2 or 1-for-5 reverse split. Since the aggregate market value of shares does not change, share price increases by the same factor. One recent example occurred on April 21, 2020 when ProShares issued a 1-for-25 reverse stock split for its Ultra Bloomberg Crude Oil Fund (UCO), a 3*x* crude oil ETF. As a result of a dramatic oversupply and a lack of storage capacity, UCO's share price dropped to $1.35. The 1-for-25 reverse stock split occurred overnight, and, by the next morning, UCO was trading at nearly $34 a share, momentarily alleviating ProShares' concern regarding potential fund collapse and liquidation.[xii]

To provide a flavor for this amusing practice, we collect the reverse split histories of four VIX ETPs—VXX, VIXY, TVIX, and SVXY. We then compute the number of days between adjacent splits, the ETP return since the last split, and the implied stopping criterion. The results are reported in Table 5. To interpret the results of the table, consider, first, the upper-left panel. VXX is a 1*x* ETN whose first day of trading was 20090130. Just 648 days later, its price had fallen –89.1%, whereupon Barclays performed a 1-for-4 reverse split. In other words, when the share price reached 10.9% of its original amount, it was stopped and recalibrated.[xiii] Another 696 days pass, the share price drops by another –81.2%, and the fund performs another 1-for-4 reverse split. The median number of days between reverse splits for VXX is 648 days. VIXY is the ETF counterpart to VXX. It has a shorter history and a median number of days between reverse splits of 888. TVIX is a 2*x* ETN on the same benchmark, SPVXSP. Since its leverage ratio is higher, we can expect that the frequency of reverse splits is higher than the 1*x* products. The results in the bottom left panel confirm this. TVIX has more splits, and the splits occur more frequently. The median number of days is 449. The most interesting comparison, however, is with UVXY. It is also a 2*x* fund and has a median number of days between reverse splits of only 207. The difference is, of course, that the issuers have different risk management policies. Credit Suisse allows TVIX to fall to a much lower price before engaging in a reverse split than ProShares does with UVXY. One possible reason for this difference is that TVIX has no options listed on it, while UVXY does. Trading in options is encumbered when share prices get to exceeding low levels.

Actual Fund Performance

Our focus now turns to actual performance. Aside from the fact that levered and inverse funds are theoretically flawed from a long-term investment/risk management perspective, there may also be problems with implementation. The question is how well issuers do at providing the levered benchmark return. To this end, we perform tracking error analyses of 35 popular funds. *Tracking error* is the difference between the daily returns of the ETP and the benchmark, $TE_t = R_{F,t} - LR_{B,t}$, where $R_{F,t}$ and $R_{B,t}$ are the daily returns of the fund and its benchmark index, respectively, and L is the leverage ratio. Note that this measurement is not risk-adjusted. It does not have to be. Whatever market price risk factors affect the ETP returns affect the benchmark returns. The mean of the tracking error is called the *tracking difference*. The *standard deviation* of the tracking error shows the effectiveness of the issuer at replication. The *t*-ratio for the null hypothesis is that the tracking difference is zero is computed by dividing the tracking difference by its standard error (i.e., the standard deviation divided by $\sqrt{T}$). The meaning of this ratio, however, extends beyond statistical significance. It is a risk-adjusted tracking difference (RATD). If two funds have the same tracking difference, the fund with the highest ratio is preferred. Because of its simple, intuitive appeal, we dub it the *RATD ratio.* With competing funds on the same benchmark, the fund with the highest RATD ratio over the evaluation period, even if it is negative, has provided the best performance. The practical reasons why RATDs vary fall into two broad categories—(a) differences between the market price of the ETP and its NAV and (b) differences between the NAV and the level of its underlying benchmark.

<u>Market price versus net asset value per share</u>

Differences between market price and net asset value per share are driven largely by the authorized participant (AP) and his interactions with the ETP issuer. Working together, the AP and the issuer ensure that the market value of the ETP's shares outstanding equals the sum of the market values of the securities/derivatives that comprise the benchmark index (i.e., the aggregate NAV of the fund). The essential elements of this arbitrage are best illustrated by narrowing the focus for the moment. Assume the fund is created using a collateralized futures position. Recall that the second-generation funds were unit leverage 1*x* and –1*x* funds on commodities or volatility. Hence, they are created using a fully collateralized futures position. The NAV of the fund equals the size of its cash (T-bills). The fund also buys 1*x* or sells –1*x* notional amount of futures. ProShares' VIXY is an example of a 1*x* product. Suppose that, on a given day, there is excess buying pressure on VIXY in the secondary market. On the other side of the customer trades is the AP. As he sells the shares of VIXY, he hedges by buying the nearby and second nearby VIX futures in the weights prescribed by the fund's benchmark, the S&P 500 VIX Short-term Futures Index (SPVXSP).[xiv] Just before the close of trading, the AP tallies up the notional amount of his futures positions and notifies ProShares of his intention to do a *creation*. The AP delivers the notional amount in cash, receives shares (or units) of VIXY, and liquidates the VIX futures position. This "in-kind" transaction ensures the market price of the ETF is exactly equal to the benchmark's indicative value. A *redemption* works exactly in reverse. If there is excessive selling pressure on VIXY on a given day, the AP delivers shares of VIXY and receives cash, which he then uses to cover his long positions. As a practical matter, the issuer creates or redeems in blocks of shares. For VIXY, a block or creation unit is 50,000 shares. The difference between the market price and the NAV is called the *premium/discount* on the fund. For products like VIXY, the premium/discount is small.[xv] The VIX futures market is deep and liquid, resulting in low trading costs. The less deep and liquid the underlying futures market, however, the greater the premium/discount. To the extent that the level of the premium/discount persists through time, it will be reflected in the *TD*. Also, APs pay a fee for each creation/redemption. Its effects are usually small. In the case of VIXY, for example, the maximum fixed fee is $250 per Creation Unit.[xvi]

The third-generation funds were the $L \neq 1$ products. They remain collateralized futures positions; however, they are either under-collateralized or over-collateralized. Again, the NAV of the fund equals the amount of cash (T-bills) it holds. In this case, however, the fund also buys or sells a notional amount of futures equal to *L* times the NAV. ProShares has two such VIX ETFs—UVXY is a 2*x* ETF based on the SPVXSP and SVXY is –1*x*.[xvii]

One final caveat regarding the market price/NAV relation is in order. If anything impedes the creation/redemption arbitrage, the ETP is in effect transformed from an open-end to a closed-end mutual fund, and the premiums/discounts can be substantial. On February 21, 2012, for example, Credit Suisse announced that it was suspending creations and redemptions of its popular 2*x* TVIX ETN. In doing so, excess demand to buy TVIX drove the share price upwards over the ensuing days to a premium of nearly 90% more than NAV per share on March 21, 2012. When the creation/redemption process resumed on March 22nd, the share price dropped by 30%, followed by another 30% drop on March 23rd.

<u>Net asset value versus benchmark</u>

Differences between the NAV and the underlying benchmark come in a variety of forms. The only explicit cost is the *total expense ratio*. It is expressed as a simple annual rate and covers the fund's operating costs and management fee. The amount of cash taken from the assets of the fund each day is usually computed as

$$\text{Cash} = ER \times \left(\frac{n}{365}\right) \times NAV_{t-1}$$

where *ER* is the expense ratio, *n* is the number of calendar days between business day $t-1$ and *t*, and NAV_{t-1} is the net value of the fund at the close on business day $t-1$. For the ETPs considered in our sample, the expense ratio ranges from 0.89% to 1.65%. On average, it amounts to a half of one basis point per day.

The remaining costs are implicit. *Indivisibility costs* arise from the fact that futures contracts have a standardized size and fractional contracts cannot be traded. The easiest way to see this in practice is by examining ETF holdings. In Table 6, we show the holdings of VIXY, UVXY, and SVXY on March 13, 2020.[xviii] The highlighted fields are directly from their files, which are available to investors each day. The other entries in the table are from other sources (listed in the table) and are used to verify ProShares' computations.

Table 6 is informative. Panel A contains the holdings of the 1*x* VIXY. As of the close on March 13, 2020, they held 663 Mar/20 VIX futures with an exposure value of $35,420,775. Since the VIX futures has a contract denomination of 1,000, the *implied* price of the Mar/20 futures is $35,420,775 / 663,000 or $53.425. We turn to the CBOE website and find that the *actual* settlement price is $53.425. The implied and actual values match exactly. The same analysis is conducted for the Apr/20 VIX futures and produces the same result. Beneath the futures contracts in Panel A is a row that begins “Net Other Assets/Cash.” Presumably this is cash or T-bills, and the amount should equal the NAV of the fund. This is confirmed by downloading a second data set from the ProShares website. It contains the entire daily NAV histories for all ProShares ETFs.[xix] The March 13, 2020 NAV for VIXY from the NAV file is $297,279,064, as indicated is a shaded cell in the table. The reported cash amount from the holding file is $297,283,619. The small difference is probably due to rounding error or interest income considerations.

At the top of the table are the weights applied to the nearby and second nearby VIX futures settlement prices on March 13, 2020, in computing SPVXSP settlement level. The weights are 10% on the Mar/20 contract and 90% on the Apr/20 contract. The ProShares positions reflect these weights as best they can. Because fractional numbers of contracts cannot be bought or sold, the actual leverage ratio will generally not be equal to the promised leverage ratio. For VIXY in Panel A, the actual leverage ratio is 0.999412*x* not the promised 1*x*. As a result, NAV returns will not exactly match the SPVXSP returns, other considerations being held constant. While the effect of this discrepancy should be randomly distributed around 0 and not affect the level of TD, it causes the standard deviation of the tracking error to increase, and the RATD ratio to fall.

The two remaining panels of Table 6 reveal similar contract indivisibility results. ProShares replication holdings, in theory, should reflect the benchmark index weights. In the case of SVXY in Panel C, the actual leverage ratio, –0.500300, is only slightly less than promised. The same is true for UVXY in Panel B. The replication holdings include a swap position. Judging by the notation, “IPATH SERIES-B S&P 500 VIX SHT-TERM FUT SWAP – GS,” the swap has Goldman Sachs as a counterparty and VXX as a reference asset. While ProShares is on the receive side of the VXX return, it is not clear what they pay. The closing price of VXX on March 13, 2020 was 43.20 according to Bloomberg. The implied price for the information in the holdings file is $67,014,447 / 1,542,184 or $43.454. The cause of the discrepancy is not clear, although the March 13, 2020 date was in an unusually high volatility period due to the Coronavirus. Similar analyses on earlier dates showed little difference between the implied and actual VXX prices. This, too, contributes to the basis risk between NAV and the benchmark.

End-of-day rebalancing costs may contribute to the basis risk between the NAV and the benchmark in at least two ways. First, a levered and inverse funds must rebalance its derivatives position at the end of day in order to deliver its promised levered return on the following day. For securities indexes, the rebalancing is accomplished using total return swaps, as pointed out by CM. Since these are OTC trades, basis risk can be minimized since the timing of the swap trade is unencumbered by exchange hours and can be negotiated when settlement prices are known. But, for futures-based indexes, the primary hedge instruments are exchange-traded futures contracts. The incremental number of futures required at the end-of-day settlement price is:

$$\left(n_{F,t} - n_{F,t-1}\right) = \left(\frac{NAV_{t-1}}{F_t}\right)\left(L^2 - L\right)R_{F,t} \tag{2}$$

where $n_{F,t}$ is the number of futures held at the close of day *t*, NAV_t is the net asset value of assets under management on day $t-1$, F_t is the futures price at the close on day *t*, and $R_{F,t}$ is the benchmark return on day *t*.[xx] These futures demands are directly proportional to the term, $L^2 - L$. For both 3*x* and –2*x* funds, for example, $L^2 - L = 6$. But, more importantly, the incremental hedging demand is in the same direction as the return of the futures index benchmark for the day, which is not fully resolved until end-of-day settlement. Since the transactions must be executed before the trading period ends, the trade prices, by definition, will have to be different from settlement prices unless by chance, and, hence, basis risk will be incurred.

Front-running is the second concern with these types of trades. Levered and inverse ETPs must execute their replication trades as closely as possible to settlement in order to minimize basis risk. The end-of-day futures demand expressed by eqn. (2), on the other hand, while not fully resolved, is almost fully known by the market well before the close. This leaves open the possibility that certain traders, knowing that the rebalancing demand at the close will be large and positive (negative), will step in ahead of issuers, buy (sell) futures, and then unwind at the close. While this type of activity exacerbates the end-of-day price movement (which should reverse in the next trading interval), it does not necessarily contribute to basis risk and, hence, tracking error.

Tracking error analysis

With the sources of tracking error identified, we turn to analyzing tracking error for 35 of the largest and most active ETPs in our four asset categories—stocks, bonds, commodities, and volatility. We proceed in two steps. First, we examine the tracking error associated with stock and bond ETFs. Recall these are characterized as carry markets since there is active arbitrage between the derivatives markets and the underlying securities markets. Table 7 contains the results. Second, we perform the same analyses on the non-carry markets—commodities and volatility. The benchmark indexes are affected by the persistent contango in the futures market and have negative expected rates of return. The results are in Table 8.

The top panel of Table 7 contains the results for the S&P 500 total return benchmark ETFs. We begin first with the –3*x* products, Direxion's SPXS and ProShares' SPXU. SPXS has an expense ratio of 1.08%, while SPXU has one of 0.91%. The tracking difference is higher for SPXU than SPXS by about half of a basis point, and the standard deviation is higher by 3.3 basis points. The RATD for SPXU is higher, meaning that it outperformed SPXS, and its absolute magnitude indicates that the tracking difference is nearly significantly different from zero from a statistical standpoint. Continuing across the rows, we show the results from a regression of the daily ETF return on the daily return of the S&P 500 index benchmark. The estimated "beta" for SPXS, –2.998, is nearly exactly its promised level of –3. The *t*-ratio is 0.70 and corresponds to the null hypothesis that the slope equals –3. It is positive, reflecting the fact that –2.998 is greater than –3, and its level does not nearly approach its cutoff level at the 5% probability level. In other words, we cannot reject the hypothesis that Direxion delivered on its promise to deliver a –3*x* product. The next column is the adjusted R-squared from the regression. At 0.998, the relation is practically perfect. Finally, the last column in the table is the holding period return over the period 20090626 through 20200313. All data series from product launch to present were collected from Bloomberg. Since we wanted to compare the different products on the same benchmark, the time series begins with the ETP with the latest launch date. Again, the results are consistent with the prediction that, if the benchmark return is expected to be positive, funds with negative leverage ratios will fail. An investment of $100 in SPXS on 20090626 is now worth 41 cents.

From a performance perspective, the other S&P 500 ETFs performed as well as the –3*x* funds. The tracking differences are all ± 1 basis point a day. According to the RATD, ProShares' 3*x* UPRO performed better than Direxion's 3*x* SPXL. On a holding period return basis, UPRO had a 1,436% return, while SPXL was 1,389%. Highly positively levered S&P 500 funds can provide extraordinary holding period returns in a slowly upward rising stock market such as that experienced during the recently-ended eleven-year bull market.

The tracking differences for all of the Russell 2000 ETFs are less than zero, and are slightly lower than their expense ratios. The standard deviations are all higher than their S&P 500 ETF counterparts. Since the Russell 2000 index is comprised of 2,000 mid-cap stocks, and, since mid-cap stocks have less deep and liquid markets, we can expect more tracking error. The noise can also be seen in the estimated regression relation. The betas for the different products are lower in an absolute value sense than they were for the comparably levered S&P 500 products. Downward biased slope coefficients are expected when the regression variables are "noisy." The results for the ETFs written on the 20+ year bond index exhibit yet even more noise. This is understandable. Relative to stock markets, bond markets are not deep, have high spreads, and are traded infrequently. The tracking differences indicate all funds perform reasonably well in relation to their stated objective. The standard deviations are much higher than the other carry benchmarks. The adjusted R-squared levels from the return regression are uniformly below 0.90.

The commodity and volatility results are reported in Table 8. Focusing on the crude oil results first, we see that many of the funds, particularly the highly levered ones, have TDs well in excess of their respective expense ratios. OILD has a TD of 4 basis points per day. The RATD shows that it is the poorest performing of the seven funds considered. The realized betas for all of the crude oil funds are less in absolute magnitude than what was promised, both in an economic and statistical sense. The adjusted R-squared levels are not close to one. While we cannot isolate the specific cause(s) of the drag, OILD is not close to replicating the performance of the benchmark. The holding period results in the last column confirm our simulation predictions. In a non-carry market in which the expected benchmark return is negative, both levered short and levered long funds will eventually fail. The holding period returns of the 3x UWT and OILU funds are both less than –92%. At the other end of the spectrum are the –3x DWT and OILD funds, both with returns less than –60%. It turns out that our prediction of failure was, sadly, accurate. On March 15, 2020, ProShares announced the liquidation of its OILU and OILD ETFs, and, on March 19, 2020, Citibank announced the acceleration of DWT and UWT. They are no longer with us.

The results for natural gas are qualitatively similar to crude. Tracking differences are mostly negative and large. The standard deviations of the tracking errors are larger for the 3*x* natural gas products than they were for the crude oil products. The highly levered 3*x* long and –3*x* funds have dismal holding period returns, –99.99% and –95.46%, respectively, as does 2*x* BOIL at –99.55%. The –2x KOLD is the single fund with a positive holding period return at 15.9%. It is also the single natural gas ETF with a positive RATD.

The final panel in Table 8 contains the results for VIX ETPs. These results are amusing in a perverse sort of way. First, note that VXX and VIXY are unlevered, lose two basis points a day, and have holding period returns of nearly –100%. This behavior was first documented by Whaley (2013) many years ago when he warned of the effects of the contango trap in the VIX futures market. That same VIX futures price behavior has persisted in the ensuing years. As expected, the 2*x* products, TVIX and UVXY, perform worse, with tracking errors exceeding six basis points a day and holding period returns of –100%. The most startling aspect of the table is the returns of the inverse products, XIV at 2,017% and SVXY at 1,906.7%. But, note that the sample ends 20180202, the Friday before XIV collapsed. On Monday, February 5, 2018, the underlying futures index, SPVXSP, spiked by 96.7%. This triggered an acceleration event and liquidation. While the SVXY was not liquidated, ProShares changed the leverage ratio from –1x to –0.5x, reducing the levered benchmark volatility by 50%. Currently, the only way to get a 100% short exposure to SPVXSP is to sell the two nearby futures and rebalance the portfolio daily to maintain the constant one-month horizon.

Conclusion

Levered and inverse funds are controversial for good reason. Unlike typical securities traded on exchanges, their expected long-run values are zero. Is this fact hidden? No. Product issuers say so in their prospectuses. The economics, mechanics, and empirics of these product structures hold the key to understanding the reasons for the litany of recent fund failures. Fund collapses were predictable from the get-go given the expected return/risk characteristics of the underlying benchmark indexes. We demonstrate this fact using Monte Carlo simulation and return/risk parameters estimated in a period before the recent spike in volatility. While compounding mechanics and dynamic rebalancing activity play roles to be sure, so does an equally important, but often neglected, aspect of levered and inverse fund instability—negative expected returns for futures-based benchmark indexes. Most of the levered and inverse product controversies in recent times are for funds whose benchmark is a futures index linked to crude oil, natural gas, or volatility. These indexes have issues in their own right. In futures markets, the demands of long hedgers often exceed those of short hedgers. Speculators step in to absorb the hedging imbalance only when the futures price is high enough to earn a satisfactory risk premium. In such futures markets, equilibrium expected benchmark returns are negative. Levering these futures-based benchmarks merely exacerbates the problem and speeds the fund's demise. Levered and inverse products are not, and cannot be, effective investment management tools.

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Endnotes

[i] See SEC press release 2019-242, *SEC proposes to modernize regulation of the use of derivatives by registered funds and business development companies* (November 25, 2019).

[ii] See Justin Baer and Paul Kierman, Levered-ETF seller urges investors to fight proposed regulation, *The Wall Street Journal* (January 31, 2020).

[iii] Popular brokers such as Fidelity, E*Trade, Vanguard, TD Ameritrade, Charles Schwab, and Merrill allow customers to open a trading account with no account minimum. In addition, all brokers do not charge any commissions for online equity and ETP trades. See https://www.fool.com/the-ascent/buying-stocks/; https://www.wsj.com/articles/the-race-to-zero-commissions-11570267802?mod=article_inline.

[iv] https://treasury.tn.gov/Portals/0/Documents/Retirement/Policies/TCRS_Investment_Policy_Approved_10-16-2018.pdf

[v] https://www.osc.state.ny.us/pension/generalpolicies.pdf

[vi] See Cheng and Madhavan (2009, p.56).

[vii] The eventual collapse of levered products is not hidden by issuers. In its prospectus for TVIX (p. 28), Credit Suisse explicitly states, "At higher ranges of volatility, there is a significant chance of a complete loss of the value of ETNs even if the performance of the applicable underlying Index is flat." At some levels, the logic here seems circular. On one hand, they know that the attraction of levered funds is their ability to create increased volatility. Yet, on the other, if the volatility of the benchmark index upon which the levered return is based becomes too high, the fund will collapse.

[viii] Most of the carry funds are replicated using OTC total return swaps agreement. The OTC market maker, in turn, uses the futures market as a key hedging instrument in managing his swap exposure.

[ix] S&P GSCI: Methodology (December 2019).

[x] S&P VIX Futures Indices: Methodology (May 2019).

[xi] The 5% stopping criterion is probably unduly conservative. We look at the actual histories of the frequency of reverse splits for VIX ETPs at the end of this section.

[xii] https://www.thestreet.com/etffocus/trade-ideas/leveraged-oil-etf-uco-undergoing-a-1-25-reverse-split

[xiii] This is in same spirit as the stopping criteria used in the simulations.

[xiv] SPVXSP has a constant one-month to expiration. The weights change each day as the futures contracts approach expiration. Nearby contracts are sold and replaced with second nearby contracts. On one each month, the index will have a single contract (with 30 days to expiration), as the futures contracts are rolled.

[xv] ETF.com estimates that 83% of ETPs traded in the U.S. trade within 1% of their NAVs and 94% trade within 2%. See Sumit Roy, *ETFs with the largest premiums and discounts*, ETF.com, March 22, 2017.

[xvi] APs may also pay a variable fee of up to 0.10% of the value of the Creation Unit.

[xvii] These ratios were unexpectedly changed to 1.5*x* and –0.50*x* on February 26, 2018.

[xviii] ProShares reports the holdings of all of its ETFs on a daily basis. Go to:

https://www.proshares.com/resources/data_downloads.html

[xix] https://www.proshares.com/resources/data_downloads.html

[xx] This equation first appeared in O'Neill and Whaley (2020).

Table 1: ETPs by issuer

		All funds				Levered and inverse funds			
		No. of		Percent of total		No. of		Percent of total	
Fund no.	Issuer	issues	$AUM	Issues	$AUM	issues	$AUM	Issues	$AUM
1	iShares	368	1,640,157,916,000	15.8%	38.8%				
2	Vanguard	80	1,121,965,492,600	3.4%	26.5%				
3	State Street SPDR	140	661,112,371,900	6.0%	15.6%				
4	Invesco	219	214,569,456,300	9.4%	5.1%	1	37,027,100	0.4%	0.1%
5	Charles Schwab	25	155,923,124,100	1.1%	3.7%				
6	First Trust	150	83,892,827,500	6.4%	2.0%				
7	VanEck	55	41,983,292,900	2.4%	1.0%				
8	WisdomTree	77	35,742,604,900	3.3%	0.8%				
9	ProShares	140	33,611,387,700	6.0%	0.8%	108	24,107,533,200	39.0%	47.6%
10	J.P. Morgan	34	32,138,565,600	1.5%	0.8%				
11	PIMCO	16	23,961,158,400	0.7%	0.6%				
12	Goldman Sachs	22	17,362,803,700	0.9%	0.4%	1	1,291,261,200	0.4%	2.6%
13	Fidelity	29	16,466,728,900	1.2%	0.4%				
14	DWS	37	15,156,061,000	1.6%	0.4%	4	167,794,600	1.4%	0.3%
15	FlexShares	29	14,311,807,600	1.2%	0.3%				
16	Direxion	97	13,010,648,400	4.2%	0.3%	80	12,467,279,500	28.9%	24.6%
17	Global X	69	10,042,180,900	3.0%	0.2%				
18	ALPS	16	8,425,033,600	0.7%	0.2%				
19	Credit Suisse	20	5,765,620,500	0.9%	0.1%	14	5,635,376,000	5.1%	11.1%
20	Pacer Financial	22	5,755,511,400	0.9%	0.1%				
…	…	…	…	…	…	…	…	…	…
140	Armor Index, Inc.	1	1,802,700	0.0%	0.0%				
		2,330	4,231,618,595,600	100.0%	100.0%	277	50,630,294,000	100.0%	100.0%

Table 2: Levered and inverse ETPs by asset class

Asset class	No. of ETPs	Total assets under management	Percent of AUM	Average dollar volume	Turnover
Equity	191	42,115,083,000	83.2%	7,133,238,098	16.9%
Commodity	26	3,106,084,200	6.1%	1,591,937,806	51.3%
Volatility	5	2,513,256,300	5.0%	2,866,578,888	114.1%
Bond	22	1,524,422,600	3.0%	153,728,289	10.1%
Real Estate	9	866,891,600	1.7%	13,355,573	1.5%
Multi-Asset	4	285,840,600	0.6%	3,679,366	1.3%
Currency	19	214,679,300	0.4%	4,419,779	2.1%
Alternatives	1	4,036,400	0.0%	48,395	1.2%
	277	50,630,294,000	100.0%		

Table 3. Simulation of life in days for levered and inverse ETPs on three popular security indexes

Panel A: SPX			CAGR: 7.87%			Volatility: 19.59%	
Leverage ratio		-3	-2	-1	1	2	3
Life in days	Minimum	439	915	2,444	5,040	2,597	853
	Median	1,644	2,795	5,040	5,040	5,040	5,040
	Prob. < 20 years	99.7%	93.8%	24.6%	0.0%	0.5%	7.6%
CLR	Minimum	-43.0%	-30.9%	-16.7%	-19.1%	-34.8%	-47.8%
	Median	-4.2%	-2.5%	-1.1%	0.8%	1.3%	1.4%
	Maximum	83.7%	50.6%	23.0%	19.4%	41.9%	68.1%
LCR-CLR	Minimum	-26.5%	-12.5%	-3.9%	0.0%	-3.3%	-10.1%
	Median	0.8%	0.4%	0.1%	0.0%	0.1%	0.4%
	Maximum	3.7%	1.9%	0.6%	0.0%	0.7%	2.1%
Prob(*CLR* **LCR* <0)		4.4%	3.2%	1.5%	0.0%	1.0%	1.9%

Panel B: RTY			CAGR: 5.70%			Volatility: 24.53%	
Leverage ratio		-3	-2	-1	1	2	3
Life in days	Minimum	285	521	2,160	5,040	2,056	659
	Median	1,460	2,633	5,040	5,040	5,040	5,040
	Prob. < 20 years	99.7%	94.6%	32.3%	0.0%	8.0%	39.1%
CLR	Minimum	-47.7%	-34.7%	-18.9%	-25.1%	-44.5%	-59.2%
	Median	-4.1%	-2.3%	-0.9%	0.4%	0.3%	-0.3%
	Maximum	126.3%	73.8%	32.4%	22.7%	49.6%	81.6%
LCR-CLR	Minimum	-50.8%	-23.5%	-7.3%	0.0%	-5.8%	-16.2%
	Median	1.3%	0.6%	0.2%	0.0%	0.2%	0.6%
	Maximum	6.8%	3.4%	1.1%	0.0%	1.1%	3.2%
Prob(*CLR* **LCR* <0)		6.3%	4.9%	2.9%	0.0%	0.9%	3.4%

Panel C: IDCOT20T			CAGR: 7.69%			Volatility: 14.11%	
Leverage ratio		-3	-2	-1	1	2	3
Life in days	Minimum	657	1,275	3,604	5,040	5,040	4,995
	Median	2,218	3,652	5,040	5,040	5,040	5,040
	Prob. < 20 years	98.7%	85.8%	5.8%	0.0%	0.0%	0.3%
CLR	Minimum	-32.1%	-22.6%	-11.9%	-12.2%	-23.0%	-32.8%
	Median	-2.6%	-1.6%	-0.7%	0.5%	0.9%	1.2%
	Maximum	45.3%	28.6%	13.5%	13.3%	28.0%	44.4%
LCR-CLR	Minimum	-8.8%	-4.3%	-1.4%	0.0%	-1.5%	-4.6%
	Median	0.4%	0.2%	0.1%	0.0%	0.1%	0.2%
	Maximum	1.8%	0.9%	0.3%	0.0%	0.3%	0.9%
Prob(*CLR* **LCR* <0)		3.2%	2.4%	1.7%	0.0%	0.4%	1.4%

Table 4. Simulation of life in days for levered and inverse ETPs on three popular futures indexes

Panel A: SPGSCLP			CAGR:	-15.51%		Volatility:	36.58%
Leverage ratio		-3	-2	-1	1	2	3
Life in days	Minimum	168	459	2,848	1,038	280	123
	Median	2,698	5,040	5,040	4,411	1,635	859
	Prob. < 20 years	87.7%	49.7%	2.9%	66.8%	98.7%	100.0%
CLR	Minimum	-67.9%	-52.4%	-30.6%	-30.8%	-52.9%	-68.5%
	Median	-1.7%	-0.1%	0.4%	-1.6%	-4.1%	-7.4%
	Maximum	178.8%	100.6%	42.6%	42.2%	99.9%	177.8%
LCR-CLR	Minimum	-86.3%	-38.9%	-11.7%	0.0%	-15.4%	-51.1%
	Median	3.0%	1.5%	0.5%	0.0%	0.5%	1.5%
	Maximum	12.7%	6.2%	2.0%	0.0%	2.1%	6.4%
Prob(*CLR* **LCR* <0)		8.4%	6.1%	4.1%	0.0%	1.8%	3.4%

Panel B: SPGSNGP			CAGR:	-35.94%		Volatility:	44.90%
Leverage ratio		-3	-2	-1	1	2	3
Life in days	Minimum	326	1,155	5,040	389	157	63
	Median	5,040	5,040	5,040	1,688	693	398
	Prob. < 20 years	48.6%	10.6%	0.0%	99.7%	100.0%	100.0%
CLR	Minimum	-75.0%	-59.4%	-35.6%	-39.8%	-65.1%	-80.6%
	Median	3.9%	4.4%	2.9%	-4.4%	-10.0%	-16.8%
	Maximum	280.0%	150.6%	60.7%	52.2%	127.0%	232.4%
LCR-CLR	Minimum	-160.6%	-71.0%	-20.9%	0.0%	-22.7%	-75.9%
	Median	3.6%	1.8%	0.6%	0.0%	0.6%	1.5%
	Maximum	20.7%	10.3%	3.4%	0.0%	3.1%	9.5%
Prob(*CLR* **LCR* <0)		8.5%	6.6%	4.6%	0.0%	1.4%	4.2%

Panel C: SPVXSP			CAGR:	-40.82%		Volatility:	67.07%
Leverage ratio		-3	-2	-1	1	2	3
Life in days	Minimum	44	107	663	258	67	41
	Median	715	2,617	5,040	1,451	506	253
	Prob. < 20 years	99.9%	88.2%	18.6%	99.8%	100.0%	100.0%
CLR	Minimum	-87.0%	-71.6%	-45.2%	-47.3%	-74.1%	-88.2%
	Median	-6.6%	-1.0%	1.3%	-5.2%	-13.0%	-23.2%
	Maximum	404.5%	206.4%	79.4%	74.3%	191.7%	370.6%
LCR-CLR	Minimum	-266.8%	-112.1%	-32.1%	0.0%	-43.1%	-147.6%
	Median	10.3%	5.1%	1.6%	0.0%	1.5%	4.2%
	Maximum	40.8%	21.5%	7.3%	0.0%	7.0%	25.4%
Prob(*CLR* **LCR* <0)		13.8%	10.3%	6.1%	0.0%	3.3%	6.7%

Table 5. Reverse split histories of 1*x* and 2*x* VIX ETPs since each fund inception

Date	Reverse ratio	Since last change: Days	Return	Stop	Date	Reverse ratio	Since last change: Days	Return	Stop
Panel A: VXX (1*x*)					Panel B: VIXY (1*x*)				
20090130	Inception				20110104	Inception			
20101109	1:04	648	-89.1%	10.9%	20130610	1:05	888	-89.1%	10.9%
20121005	1:04	696	-81.2%	18.8%	20160725	1:05	1,141	-81.2%	18.8%
20131108	1:04	399	-64.4%	35.6%	20170717	1:04	357	-64.4%	35.6%
20160809	1:04	1,005	-81.2%	18.8%					
20170823	1:04	379	-67.4%	32.6%					
Mean		625.4	-76.7%	23.3%	Mean		795.3	-78.2%	21.8%
Median		648	-81.2%	18.8%	Median		888	-81.2%	18.8%
Panel C: TVIX (2*x*)					Panel D: UVXY (2*x*)				
20101130	Inception				20111004	Inception			
20121221	1:10	752	-99.1%	0.9%	20120308	1:06	156	-85.2%	14.8%
20130830	1:10	252	-79.7%	20.3%	20120907	1:10	183	-88.6%	11.4%
20150623	1:10	662	-96.6%	3.4%	20130610	1:10	276	-82.4%	17.6%
20160809	1:25	413	-87.3%	12.7%	20140124	1:04	228	-70.2%	29.8%
20170316	1:10	219	-83.7%	16.3%	20150520	1:05	481	-88.7%	11.3%
20180608	1:10	449	-88.3%	11.7%	20160725	1:05	432	-85.4%	14.6%
20191202	1:10	542	-83.8%	16.2%	20170112	1:05	171	-79.8%	20.2%
					20170717	1:04	186	-73.4%	26.6%
Mean		469.9	-88.4%	11.6%	Mean		264.1	-81.7%	18.3%
Median		449	-87.3%	12.7%	Median		207	-83.8%	16.2%

Table 6. Holdings and NAVs of ProShares VIX ETFs at the close on March 13, 2020

Futures	Settlement price	SPVXSP weight	Source
VIX futures 20200318	53.425	10.000%	CBOE
VIX futures 20200415	43.900	90.000%	CBOE

ETN	Closing price	Source
VXX	43.200	Bloomberg

Source of ETF holdings: https://www.proshares.com/resources/data_downloads.html

Panel A: VIXY ProShares VIX Short-Term Futures ETF (1x)

Holdings	Shares/ contracts	Exposure value	Market value	Implied price	Index weights
VIX futures 20200318	663	35,420,775		53.425	10.009%
VIX futures 20200415	5,961	261,687,900		43.900	89.991%
Net Other Assets / Cash	297,283,619		297,283,619		
Total exposure value		297,108,675			
Cash/NAV/Difference		297,283,619	297,279,064	4,555	
Leverage ratio		0.999412			

Panel B: UVXY ProShares Ultra VIX Short-Term Futures ETF (1.5x)

Holdings	Shares/ contracts	Exposure value	Market value	Implied price	Index weights
VIX futures 20200318	2,318	123,839,150		53.425	10.018%
VIX futures 20200415	20,820	913,998,000		43.900	89.982%
VXX*	1,542,184	67,014,447		43.454	
Net Other Assets / Cash	736,727,443		736,727,443		
Total exposure value		1,104,851,597			
Cash/NAV/Difference		736,727,443	736,734,249	-6,805	
Leverage ratio		1.499675			

VXX* is IPATH SERIES-B S&P 500 VIX SHT-TERM FUT SWAP - GS

Panel C: SVXY ProShares Short VIX Short-Term Futures ETF (-0.5*x*)

Holdings	Shares/ contracts	Exposure value	Market value	Implied price	Index weights
VIX futures 20200318	-609	-32,535,825		53.425	9.935%
VIX futures 20200415	-5,521	-242,371,900		43.900	90.065%
Net Other Assets / Cash	549,485,607		549,485,607		
Total exposure value		-274,907,725			
Cash/NAV/Difference		549,485,607	549,508,907	-23,300	
Leverage ratio		-0.500300			

Table 7. Analysis of tracking errors and daily returns of ETPs benchmarked to total return of security indexes

		Tracking error summary			Return summary			
		Tracking	Standard	RATD		H_0: β=L		
Fund	*ER*	difference	deviation	ratio	β	t-ratio	Adj. R^2	*HPR*
Panel A: S&P 500								
Sample period:		20090626	20200313	No. of obs.:	2,694			267.87%
SPXS, Direxion, -3x ETF	1.08%	0.002%	0.183%	0.564	-2.998	0.70	0.998	-99.59%
SPXU, ProShares, -3x ETF	0.91%	0.007%	0.216%	1.615	-2.972	6.96	0.997	-99.53%
SDS, ProShares, -2x ETF	0.89%	0.004%	0.109%	2.070	-1.989	5.31	0.999	-96.33%
SH, ProShares, -1x ETF	0.89%	0.002%	0.064%	1.265	-0.997	2.69	0.998	-78.44%
SSO, ProShares, 2x ETF	0.90%	-0.008%	0.116%	-3.710	1.983	-7.78	0.998	726.41%
SPXL, Direxion, 3x ETF	1.02%	-0.014%	0.187%	-3.916	2.981	-5.29	0.998	1389.01%
UPRO, ProShares, 3x ETF	0.92%	-0.013%	0.212%	-3.242	2.970	-7.41	0.998	1435.59%
Panel B: Russell 2000								
Sample period:		20081106	20200313	No. of obs.:	2,856			175.09%
TZA, Direxion, -3x ETF	1.11%	-0.009%	0.367%	-1.369	-2.918	18.84	0.994	-99.91%
TWN, ProShares, -2x ETF	0.95%	-0.005%	0.268%	-0.937	-1.948	16.26	0.992	-98.19%
RWM, ProShares, -1x ETF	0.95%	-0.002%	0.127%	-0.904	-0.976	15.49	0.993	-81.72%
UWM, ProShares, 2x ETF	0.95%	-0.008%	0.280%	-1.614	1.940	-17.97	0.992	230.86%
TNA, Direxion, 3x ETF	1.14%	-0.017%	0.414%	-2.154	2.886	-24.21	0.992	118.89%
Panel C: 20-year T-bond								
Sample period:		20100122	20200313	No. of obs.:	2,550			132.29%
TMV, Direxion, -3x ETF	1.04%	0.000%	0.889%	0.007	-2.845	7.77	0.889	-97.54%
TBT, ProShares, -2x ETF	0.90%	-0.003%	0.595%	-0.208	-1.868	9.94	0.886	-90.24%
TBF, ProShares, -1x ETF	0.92%	-0.001%	0.297%	-0.170	-0.940	9.05	0.887	-65.44%
UBT, ProShares, 2x ETF	0.95%	-0.010%	0.580%	-0.850	1.918	-6.27	0.894	242.25%
TMF, Direxion, 3x ETF	1.05%	-0.014%	0.892%	-0.797	2.844	-7.82	0.888	382.75%

Table 8. Analysis of tracking errors and daily returns of ETPs benchmarked to futures indexes

Fund	*ER*	Tracking error summary: Tracking difference	Standard deviation	RATD ratio	Return summary: β	H_0: β=L t-ratio	Adj. R^2	*HPR*
Panel A: Crude oil								
Sample period:		20170328	20200313	No. of obs.:	745			-36.15%
DWT, Citigroup, -3x ETN	1.50%	-0.026%	1.674%	-0.431	-2.859	5.19	0.937	-60.37%
OILD, ProShares, -3x ETF	0.49%	-0.040%	1.676%	-0.649	-2.830	6.28	0.936	-63.26%
SCO, ProShares, -2x ETF	0.95%	-0.022%	1.110%	-0.549	-1.878	6.80	0.937	-26.21%
USO, USCF, 1x ETF	0.73%	0.010%	0.557%	0.485	0.944	-6.24	0.936	-30.68%
UCO, ProShares, 2x ETF	0.95%	0.009%	1.110%	0.230	1.849	-8.60	0.937	-69.29%
UWT, Citigroup, 3x ETN	1.50%	-0.012%	1.767%	-0.181	2.732	-9.70	0.929	-92.81%
OILU, ProShares, 3x ETF	0.49%	-0.006%	1.759%	-0.091	2.719	-10.29	0.930	-92.20%
Panel B: Natural gas								
Sample period:		20120208	20200313	No. of obs.:	2,037			-83.51%
DGAZ, Credit Suisse, -3x ETN	1.65%	-0.043%	2.190%	-0.878	-2.702	17.01	0.921	-95.46%
KOLD, ProShares, -2x ETF	0.95%	0.008%	1.539%	0.243	-1.727	23.38	0.915	15.90%
UNG, USCF, 1x ETF	1.28%	0.000%	0.732%	-0.004	0.918	-13.61	0.920	-82.54%
BOIL, ProShares, 2x ETF	0.95%	-0.047%	1.545%	-1.358	1.722	-23.81	0.914	-99.55%
UGAZ, Credit Suisse, 3x ETN	1.65%	-0.041%	2.157%	-0.847	2.692	-17.93	0.924	-99.99%
Panel C: Volatility								
Sample period:		20111005	20180202	No. of obs.:	1,593			-99.74%
XIV, Credit Suisse, -1x ETN	1.65%	-0.016%	1.367%	-0.468	-0.868	17.90	0.897	2016.61%
SVXY, ProShares, -1x ETF	1.38%	-0.019%	1.363%	-0.541	-0.873	17.15	0.898	1906.65%
VXX, Barclays, 1x ETN	0.89%	-0.017%	1.311%	-0.524	0.871	-18.36	0.906	-99.76%
VIXY, ProShares, 1x ETF	0.87%	-0.017%	1.310%	-0.524	0.875	-17.72	0.906	-99.76%
TVIX, Credit Suisse, 2x ETN	1.65%	-0.092%	3.123%	-1.175	1.618	-24.25	0.869	-100.00%
UVXY, ProShares, 2x ETF	1.65%	-0.066%	2.643%	-1.002	1.737	-18.60	0.905	-100.00%